Pre-Calculus Polynomial Worksheet

For #1-4, use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function. Then use this end behavior to match the polynomial function with its graph.

1. \( f(x) = -x^3 + x^2 + 2x \)
2. \( f(x) = x^6 - 6x^4 + 9x^2 \)
3. \( f(x) = x^5 - 5x^3 + 4x \)
4. \( f(x) = -x^4 - 1 \)

For #5-6, find the zeros for each polynomial function and give the multiplicity of each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

5. \( f(x) = -2(x - 1)(x + 2)^2(x + 5)^3 \)
6. \( f(x) = x^3 - 5x^2 - 25x + 125 \)

For #7-8,

a. Use the Leading Coefficient Test to determine the graph’s end behavior.

b. Determine whether the graph has y-axis symmetry, origin symmetry, or neither.

c. Graph the function.

7. \( f(x) = 4x - x^3 \)
8. \( f(x) = -x^4 + 6x^3 - 9x^2 \)

Divide using long division.

9. \( (4x^3 - 3x^2 - 2x + 1) \div (x + 1) \)
10. \( (10x^3 - 26x^2 + 17x - 13) \div (5x - 3) \)
11. \( (4x^4 + 6x^3 + 3x - 1) \div (2x^2 + 1) \)

Divide using synthetic division.

12. \( (3x^4 + 11x^3 - 20x^2 + 7x + 35) \div (x + 5) \)
13. \( (3x^4 - 2x^3 - 10x) \div (x - 2) \)
14. \( (x^2 - 6x - 6x^3 + x^4) \div (6 + x) \)
15. Given \( f(x) = 2x^3 - 7x^2 + 9x - 3 \), use the Remainder Theorem to find \( f(-13) \).

For #16-22, factor the given polynomial completely, or state that the polynomial is prime.

16. \( 64 - x^2 \)  
17. \( 3x^4 - 9x^3 - 30x^2 \)  
18. \( 16x^2 - 40x + 25 \)  
19. \( y^3 - 8 \)  
20. \( x^2 + 16 \)  
21. \( 3x^4 - 12x^2 \)  
22. \( (x^2 - 4)(x^2 + 3)^\frac{1}{2} - (x^2 - 4)(x^2 + 3)^\frac{3}{2} \)

For #23-27, solve each polynomial equation.

23. \( 2x^2 - 11x + 5 = 0 \)  
24. \( (3x + 5)(x - 3) = 5 \)  
25. \( (x - 3)^2 - 24 = 0 \)  
26. \( 2x^3 - 5x^2 + x + 2 = 0 \), given 2 is a zero.  
27. \( 3x^3 + 7x^2 - 22x - 8 = 0 \) given \(-\frac{1}{3}\) is a root.

For #28-30,

a. List all possible rational roots or rational zeros.

b. Use synthetic division to test the possible rational roots or zeros and find an actual root or zero.

c. Use the quotient from part (b) to find all the remaining zeros or roots.

28. \( f(x) = x^3 + 3x^2 - 4 \)  
29. \( f(x) = 6x^3 + x^2 - 4x + 1 \)  
30. \( f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4 \)  

31. Find all the zeros of the polynomial function and write the polynomial as a product of linear factors. \( f(x) = x^4 - 6x^3 + x^2 + 24x + 16 \)