Math Virtual Learning

Calculus AB

Tuesday, April 14, 2020

Lesson: Tuesday, April 14, 2020
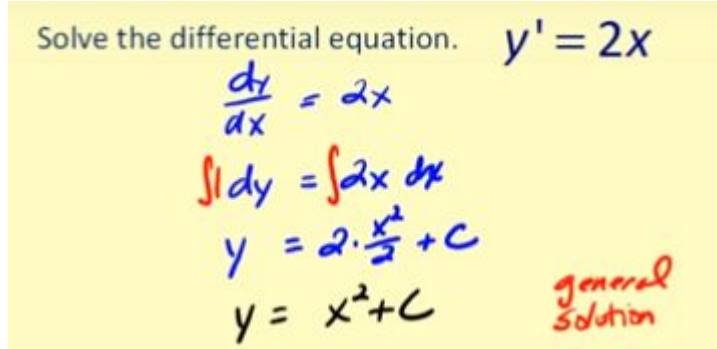
Objective/Learning Target:
I can sketch a family of functions (slope field) to represent the general solution of a differential equation
I can find a particular solution to a differential equation

Introduction

Use the video to start looking at sketches of slope fields: <u>Watch Video</u>

A slope field allows us to sketch a solution curve to an integration problem.
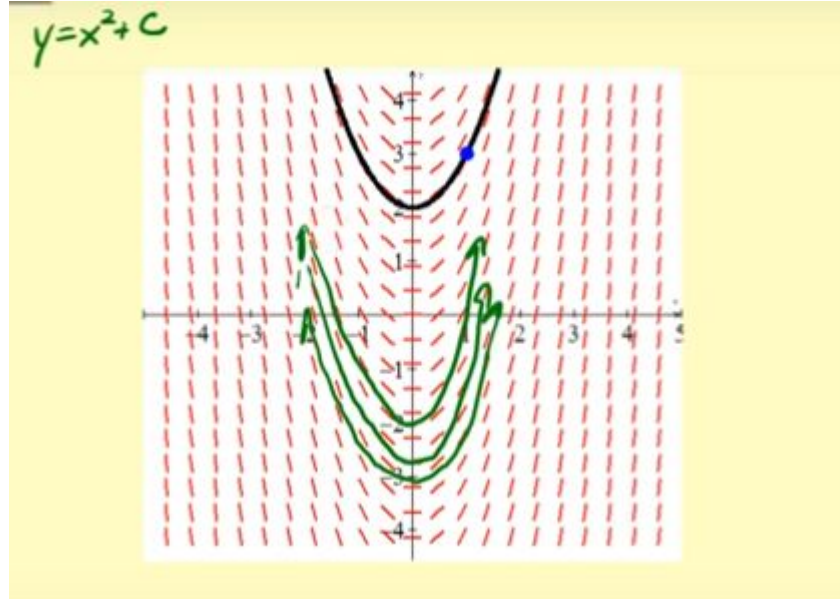
Solve the differential equation. $y' = 2x$

$$\frac{dy}{dx} = 2x$$

$$\int 1\, dy = \int 2x\, dx$$

$$y = 2 \cdot \frac{x^2}{2} + C$$

$$y = x^2 + C \qquad \text{general solution}$$

Notice the problem above, remember the "c" is called the constant of integration. This is a general solution because we don't know the value of c.

Introduction

Notice, any of the parabolas sketched could represent the equation. We can draw the slope of the tangent at all points to represent any solution.

$$y = x^2 + C$$

Sketch a Slope Field

Given: $\frac{dy}{dx} = 2x$.

Remember, this represents the slope of the tangent at any given point. Let's create a table for all of these tangents.

Hint: this equation is only in terms of x, we will not need the y-coordinate when evaluating.

Check the next slide for answers!

Using the differential equation above, find the slope at each of the following points.

| Point	Slope of tangent at the point	Point	Slope of tangent at the point
$(-2, 2)$	$\frac{dy}{dx} = 2x$ $\frac{dy}{dx} = 2(-2) = -4$	$(0, -1)$	
$(-2, 0)$		$(0, 0)$	
$(-2, -1)$		$(1, 1)$	
$(-1, 2)$		$(1, 2)$	
$(-1, 1)$		$(1, -2)$	
$(-1, 0)$		$(2, 1)$	
$(-1, -2)$		$(2, 0)$	
$(0, 1)$		$(2, -1)$	

Sketch a Slope Field

Is this your table of values?

| Point	Slope of toothpick tangent	Point	Slope of toothpick tangent
$(-2, 2)$	-4	$(0, -1)$	0
$(-2, 0)$	-4	$(0, 0)$	0
$(-2, -1)$	-4	$(1, 1)$	2
$(-1, 2)$	-2	$(1, 2)$	2
$(-1, 1)$	-2	$(1, -2)$	2
$(-1, 0)$	-2	$(2, 1)$	4
$(-1, -2)$	-2	$(2, 0)$	4
$(0, 1)$	0	$(2, -1)$	4

Sketch a Slope Field

For each of the ordered pairs above, sketch a mini tangent at that point with the given slope. Can you fill in the rest of the points based on patterns you see?
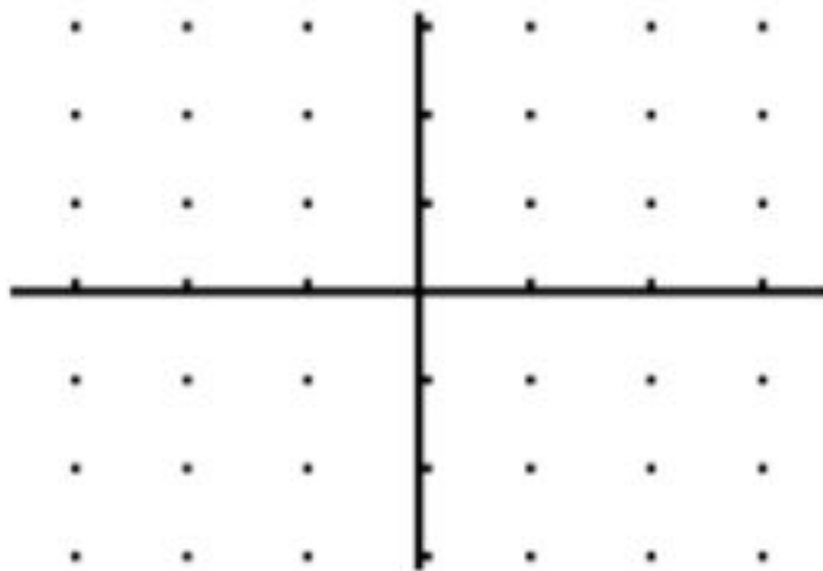
Sketch a curve that follows those slopes. How many curves are possible? How do you know?

Sketch a Slope Field- Answer
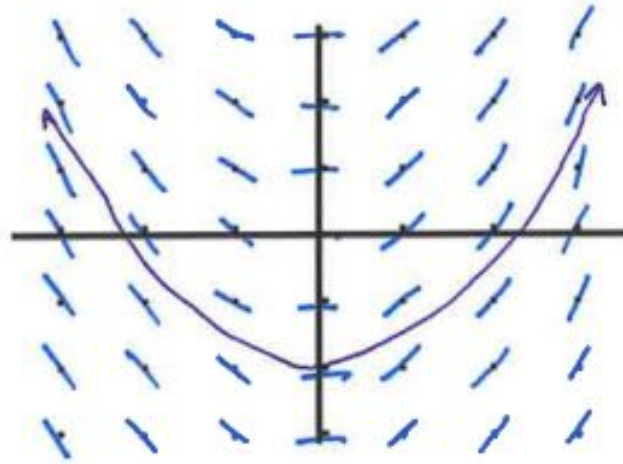
Slope in this diff. eq. depends only on x!

Slope field shows the family of solution curves.

4. Sketch a curve that follows those slopes. How many curves are possible? How do you know?

Infinitely many curves because the same curve could be shifted up or down.

Summary

Important Ideas:

Slope fields are a graphical representation of a differential equation that allow us to visualize the family of solution curves.

Making a slope field:
- calculate slope at various ordered pairs
- plot slopes using short line segments

A solution curve will follow the trend of the slopes and must pass through the initial condition if given.
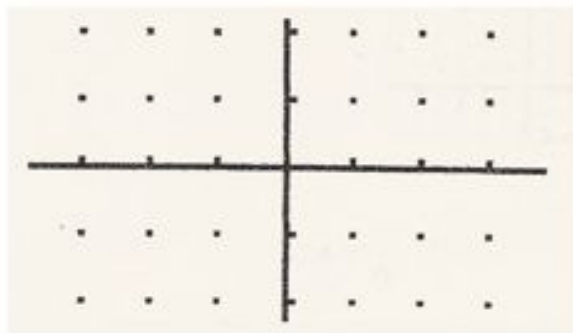
If $\frac{dy}{dx}$ is undefined, do NOT draw a slope there.
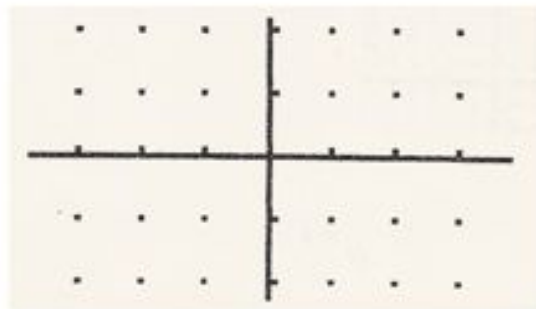
Practice

Check Your Understanding!

1. Draw a slope field for each differential equation. Each tick mark is one unit.

$$\frac{dy}{dx} = -y$$
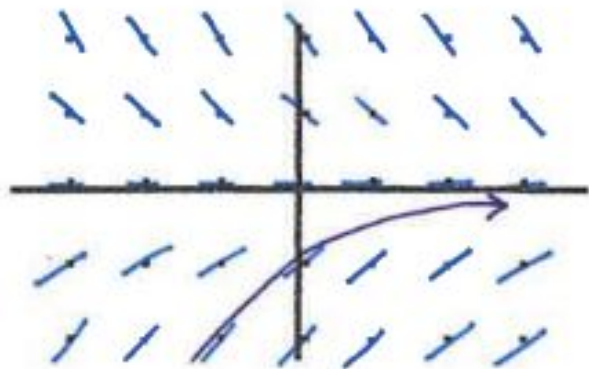
$$\frac{dy}{dx} = \frac{x}{y}$$

2. Sketch the particular solution to $\frac{dy}{dx} = -y$ that passes through $(0, -1)$ on your slope field above.
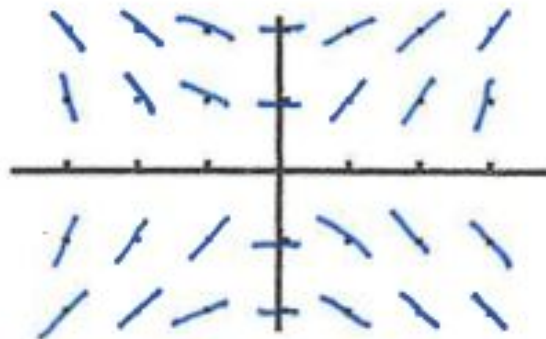
Practice Answers

1. Draw a slope field for each differential equation. Each tick mark is one unit.

$$\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{x}{y}$$

2. Sketch the particular solution to $\frac{dy}{dx} = -y$ that passes through $(0, -1)$ on your slope field above.

More Videos

Khan Academy [Slope Field from Equation](#)

Khan Academy [Equation from Slope Field](#)

Khan Academy [Forming a Slope Field](#)

More Practice

More [Practice with Answers](#)

Even [More Practice with Answers](#)

[AP Practice with Answers](#)

Book Practice:

Pg. 410: 38, 42, 47, 53-56

The practice problems and solutions were taken from the [Calc Medic online site](#).