Math Virtual Learning

Calculus AB

Wednesday, April 15, 2020

Lesson: Wednesday, April 15, 2020

Objective/Learning Target:

I can use separation of variables to find a general solution to a differential equation

We've already seen that slope fields give us a visual representation of the solutions to a differential equation. But how do we find these solutions by hand? And how does the "+C" affect the solution? Today we will explore these questions.

Remember- we are asking ourselves the question, "what did I take the derivative of to get this differential equation?"

The slope field gives us a visual of the differential equation. Now, we will find the answer to what we took the derivative of.

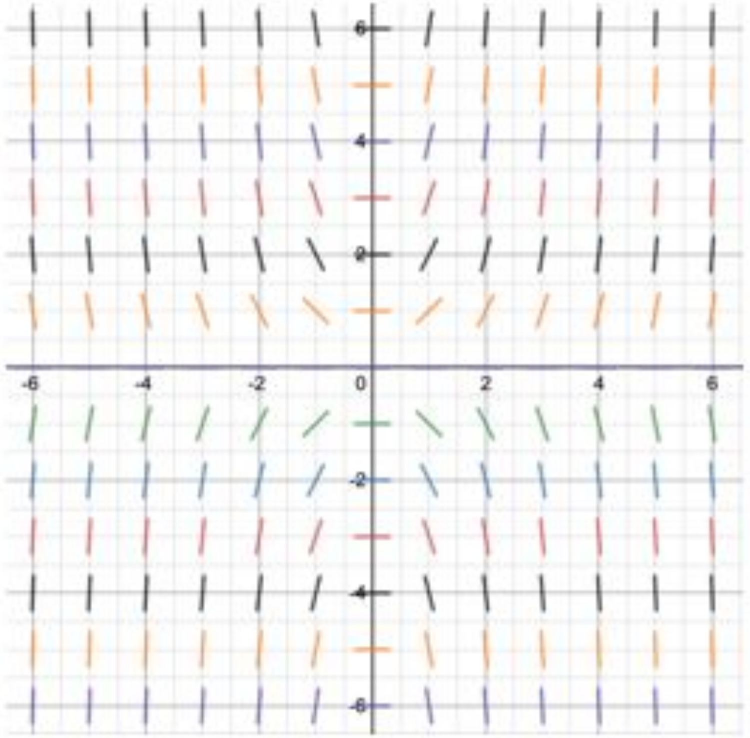
Introduction

1. Let $\frac{dy}{dx} = xy$. The slope field is shown below.
   a. Describe the slopes of the tangent lines in each quadrant.

   b. Sketch three possible solution curves to this differential equation.

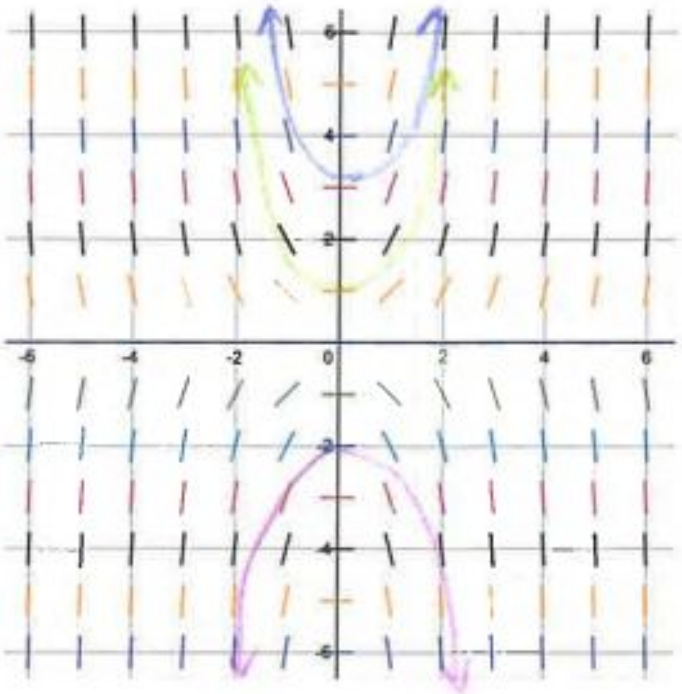
   c. Describe the general shape of a solution curve.

   d. How do the solution curves differ? Why do you think this is?



Introduction Answers

1. Let $\frac{dy}{dx} = xy$. The slope field is shown below.



   a. Describe the slopes of the tangent lines in each quadrant.

   Q1 – pos Q2 – neg
   Q3 – pos Q4 – neg

   b. Using three different colored pencils, sketch three possible solution curves to this differential equation.

   c. Describe the general shape of a solution curve.

   looks like a parabola
   y-axis symmetry

   d. How do the solution curves differ? Why do you think this is?

   Some face up, some face down
   This is related to having a positive or negative coefficient.

Finding the general solution

a. Re-write $\frac{dy}{dx} = xy$ so all the terms with *y* are on the left side and all the terms with *x* are on the right side.

Finding the general solution

a. Re-write $\frac{dy}{dx} = xy$ so all the terms with y are on the left side and all the terms with x are on the right side.

Separate

$$\frac{dy}{y} = x\,dx$$

Finding the general solution

b. Take the integral of both sides.

Finding the general solution

Integrate b. Take the integral of both sides.

$$\int \frac{dy}{y} = \int x\,dx \qquad \ln|y| + C_1 = \frac{x^2}{2} + C_2$$

$$\ln|y| = \frac{x^2}{2} + C_2 - C_1$$

$$\ln|y| = \frac{x^2}{2} + C$$

Notice we can use +c on both sides of the equation. But they are like terms and can be combined to one term.

Finding the general solution

c. Get y by itself. How does the algebra support what you found in 1d?

Finding the general solution

isolate

$y = \pm C e^{x^2/2}$

c. Get *y* by itself. How does the algebra support what you found in 1d?

$\ln|y| = \frac{x^2}{2} + c$

$|y| = e^{\frac{x^2}{2} + c}$

$y = \pm e^{x^2/2 + c}$

The ± explains why some open up + others open down

Important Ideas

Important Ideas:

To solve a first-order differential equation of the form $\frac{dy}{dx} = f(x)g(y)$, use separation of variables.

① Separate (Move all terms w/ y to one side and all terms w/ x to the other)

② Integrate (Don't forget +C!)

③ Isolate (Get y by itself, watch out for ± solutions)

More Practice

1. Use separation of variables to find the general solution to the differential equation.
   a. $\frac{dy}{dx} = 3y$
   b. $\frac{dy}{dx} = \frac{x}{y}$

2. What is the general solution to the differential equation $\frac{dy}{dx} = \frac{\cos x\, e^{\sin x}}{\cos y}$?

3. What is the general solution to the differential equation $\frac{dy}{dx} = y^2 - y^2 \sin x$?

More Practice Answers

1. Use separation of variables to find the general solution to the differential equation.

a. $\frac{dy}{dx} = 3y$

$\int \frac{dy}{y} = \int 3\,dx$

$\ln|y| = 3x + c$

$|y| = e^{3x+c}$

$y = \pm e^{3x+c} = \pm ce^{3x}$

b. $\frac{dy}{dx} = \frac{x}{y}$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

$y^2 = x^2 + c$

$y = \pm\sqrt{x^2 + c}$

More Practice Answers

2. What is the general solution to the differential equation $\frac{dy}{dx} = \frac{\cos x \, e^{\sin x}}{\cos y}$?

$$\int \cos y \, dy = \int \cos x \, e^{\sin x} \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\sin y = e^{\sin x} + C$$

$$y = \sin^{-1}\left(e^{\sin x} + C\right)$$

More Practice Answers

3. What is the general solution to the differential equation $\frac{dy}{dx} = y^2 - y^2 \sin x$?

$$\frac{dy}{dx} = y^2(1 - \sin x)$$

$$\int \frac{dy}{y^2} = \int (1 - \sin x)\, dx$$

$$\frac{-1}{y} = \cos x + C$$

$$y = \frac{-1}{\cos x + C}$$

Extra Practice

Practice with answers

Textbook Section 6.3- pg. 429: 4, 6, 16, 20

Lesson Developed from resources found on Calc Medic