Math Virtual Learning

AP Stats

Power Day 3

April 17th, 2020

Lesson: April 17th, 2020

Objective/Learning Target:
Students will extend their understanding of power to include the effects of significance levels.

Review Questions

The first page of the worksheet contains the review problems that you will see on the next page. Open it up and follow along.

Link: worksheet

Review Questions

We have been testing the following hypotheses:

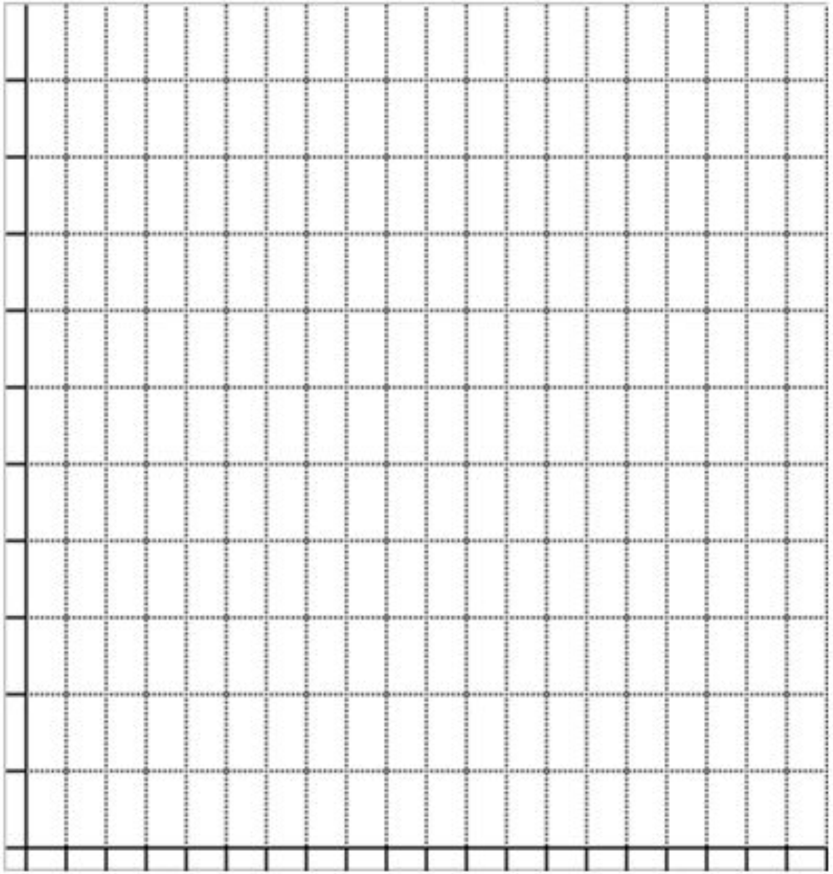
$$H_O : p = 0.20$$
$$H_A : p < 0.20$$

where p = the proportion of all boxes with the voucher

The alternative hypothesis specifies any value less than 0.20. The table below lists possible alternative values, along with an estimate of the power of the 65 box test with $\alpha = .05$ against each alternative. These estimates of the power were obtained by simulating 1000 trials of 65 box tests (using $\alpha = .05$) for each alternative.

H_A: p = ___	Power against H_A (estimated)
.19	.066
.18	.080
.17	.129
.16	.150
.15	.226
.14	.292
.13	.358
.12	.464
.11	.579
.10	.673
.09	.775
.08	.846
.07	.921
.06	.963
.05	.982
.04	.994
.03	1
.02	1
.01	1
0	1

On the axes below, construct a scatter plot of the data in the table at left. Put p on the x-axis and Power on the y-axis. Connect the dots with a line.

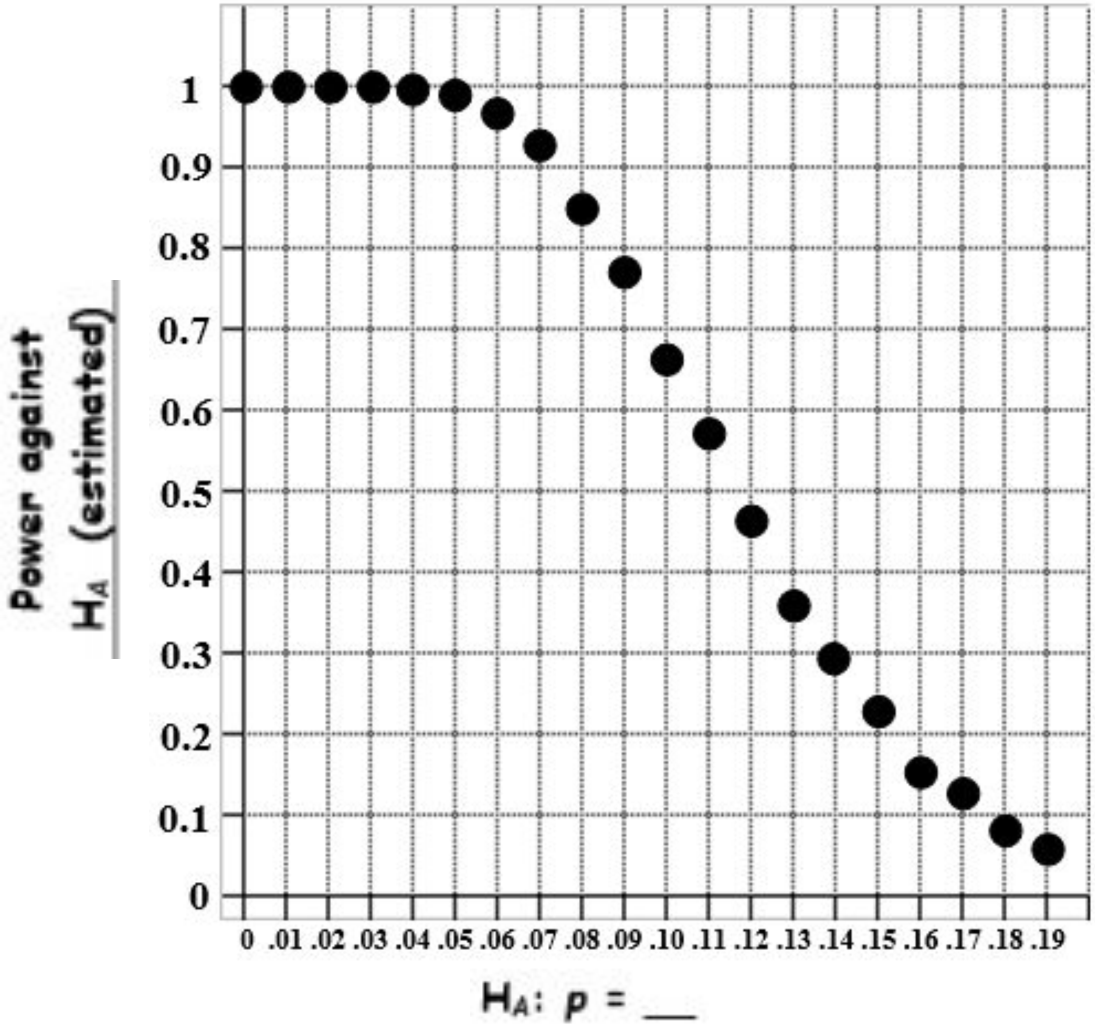


Review Questions

Comment on how the distance between $p = 0.2$ and the alternative values of p affects the power of the 65 box test with $\alpha = .05$. Specifically, as the distance increases, how does the power of the test change?

Answers

Note that values on the right are closer to the null value. As we move to the left the difference between the null and alternative values becomes larger.



Answers

Comment on how the distance between $p = 0.2$ and the alternative values of p affects the power of the 65 box test with $\alpha = .05$. Specifically, as the distance increases, how does the power of the test change?

As the distance between $p = 0.2$ and the alternative value of p increases, the power of the 65 box test increases. The closer that the alternative value of p is to the null hypothesis of $p = 0.2$, the harder it will be to determine that the company is cheating and that is why the power is low for values of p close to 0.2.

Power and Significance level

This activity works with the same scenario that we have been looking at the past several days. We will continue to work with a sample size of n=65 boxes. Our hypotheses will continue to be:

$$H_O : p = 0.20$$
$$H_A : p < 0.20$$

where p = the proportion of all boxes with the voucher

The only difference is this time we are going to vary the significance level and see how that changes the power of the test.

What effect do you predict the significance level is going to have on the power?

Effects of significance level

Statisticians are interested in the power of their tests of significance. Knowing how much power a test has against a certain alternative gives them an idea of how likely it is for their significance test to reject the null hypothesis correctly if a certain alternative is true. You have seen that the sample size and the distance between the hypothesized value and the alternative value of p affect the power of a test. There is one more factor that affects the power.

It was assumed the students used a significance level of $\alpha = .05$ in performing their 65 box test. But what if they used a significance level of $\alpha = .01$? $\alpha = .10$? You will investigate below how changing the significance level affects power in this section.

Effects of significance level

Earlier, you discovered that in a 65 box test using $\alpha = .05$, the students would conclude the company was cheating if they obtained 7 or fewer boxes with vouchers. Use trial and error with One Proportion z-test on your calculator to find the upper limit of voucher boxes students would need to find in order to conclude the company is cheating for the other significance levels. Fill in the table.

Significance Level (α)	.01	.05	.10
Upper Limit of voucher boxes in order to conclude cheating		7	

Effects of significance level

Pretend the company is cheating—they are putting the voucher in only 10% of all boxes. Under this assumption, 65 boxes from the population were randomly sampled in 1000 separate instances. In each of the 1000 trials, the number of voucher boxes obtained was recorded. The results from the 1000 random samples taken from the 10% voucher population are recorded in the table.

Number of Voucher Boxes	Frequency
0	1
1	6
2	20
3	68
4	106
5	162
6	173
7	160
8	114
9	77
10	64
11	26
12	13
13	8
14	1
15	0
16	1

Effects of significance level

. Based on the results of the 1000 random samples, calculate estimates of the power of the 65 box test against an alternative of $p = 0.10$ for the different significance levels. Fill in the table.

Significance Level (α)	.01	.05	.10
Estimate of POWER against $p = 0.10$			

Effects of significance level

Comment on how the significance level of a test of significance affects the power of the 65 box test against an alternative of $p = 0.10$. Specifically, as the significance level increases, how does the power change?

THE THREE FACTORS AFFECTING POWER

As you have seen, there are three factors that affect the power of a test of significance. They are

I. The sample size (n).

II. The true value of the population characteristic of interest.

III. The significance level (α).

THE THREE FACTORS AFFECTING POWER

21. To summarize the effect these three factors have on power, fill in the table below.

	What happens to the Power?
When the sample size increases…	
When the distance between the hypothesized and alternative values of *p* increases…	
When the significance level increases…	

In general, statisticians determine what alternative value it is important for them to detect, and select a sample size for their study that gives them the power they desire against that alternative. Thus, a common way for a statistician to adjust the power against a particular alternative is to adjust the sample size.

ANSWERS!

20. Earlier, you discovered that in a 65 box test using $\alpha = .05$, the students would conclude the company was cheating if they obtained 7 or fewer boxes with vouchers. Use trial and error with One Proportion z-test on your calculator to find the upper limit of voucher boxes students would need to find in order to conclude the company is cheating for the other significance levels. Fill in the table.

5 boxes give a *P*-value less than .01 and 8 boxes give a *P*-value less than .10.

Significance Level (α)	.01	.05	.10
Upper Limit of voucher boxes in order to conclude cheating	5	7	8

ANSWERS!

21. Based on the results of the 1000 random samples, calculate estimates of the power of the 65 box test against an alternative of $p = 0.10$ for the different significance levels. Fill in the table.

To get the numbers in the table below, I calculated the proportion of the 1000 random samples on the previous page that had less than or equal to 5, 7, and 8 boxes, respectively.

Significance Level (α)	.01	.05	.10
Estimate of POWER against p = 0.10	0.363	0.696	0.810

ANSWERS!

. Comment on how the significance level of a test of significance affects the power of the 65 box test against an alternative of $p = 0.10$. Specifically, as the significance level increases, how does the power change?

As the significance level increases, the power of the 65 box test against the alternative of $p = 0.10$ increases.

ANSWERS!

	What happens to the Power?
When the sample size increases...	Increases
When the distance between the hypothesized and alternative values of *p* increases...	Increases
When the significance level increases...	Increases