Inspiring Greatness

INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

AP Calculus AB

April 20, 2020

Lesson: Monday, April 20, 2020

Objective/Learning Target:

- I can calculate areas between curves expressed as functions of x
- I can find intersections of curves to determine the limits of integration
- I can use a sum of multiple definite integrals when curves intersect at more than two places

Review: Definite Integral Practice

Introduction

In 2012, the New York Times claimed that the wealthiest 1% of the people in the country received just under a fifth of the country's pretax income. Today we'll explore how Calculus can help us understand this income and wealth inequality.

1. The Lorenz curve $L(x)$ gives the proportion of the total income earned by the lowest proportion x of the population. For example, $L(0.90) = 0.55$ would mean the poorest 90% earn 55% of the total income.
    a. Interpret $L(0.25) = 0.10$.

    b. If everyone earned exactly the same amount of money what would $L(0.3)$ be? What would $L(0.65)$ be?

Answer

1. The Lorenz curve $L(x)$ gives the proportion of the total income earned by the lowest proportion x of the population. For example, $L(0.90) = 0.55$ would mean the poorest 90% earn 55% of the total income.
   a. Interpret $L(0.25) = 0.10$.

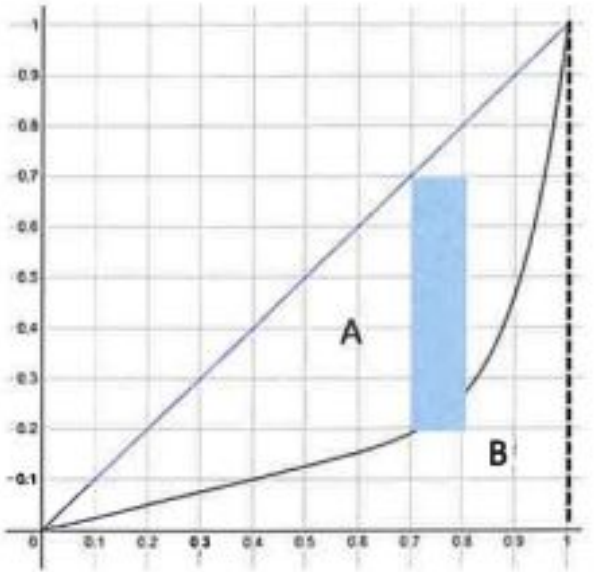
      The poorest 25% earn 10% of the total income

   b. If everyone earned exactly the same amount of money what would $L(0.3)$ be? What would $L(0.65)$ be?

      $L(0.3) = 0.3$ $L(0.65) = 0.65$

Introduction

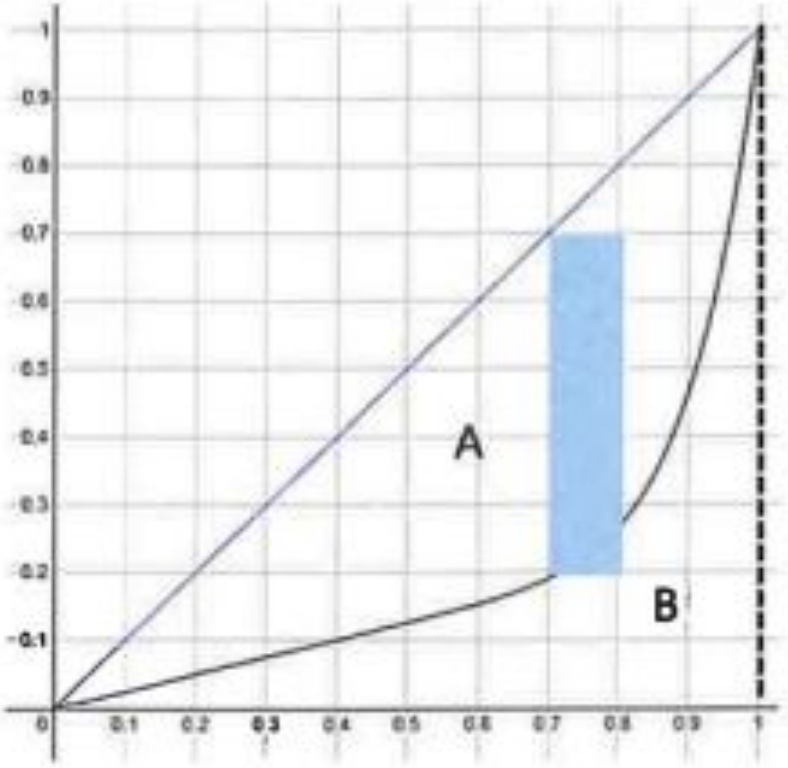
2. The curve $y = x$ represents total income equality. The Lorenz function for a particular country is given by $L(x) = 0.25 + 0.75x^{11}$. What does the area between these functions, region A, tell you about a country's wealth distribution?



Introduction

3. To approximate the area of region A, split the region into 10 rectangles. One has been shown as an example.
    a. What is the width of each rectangle?

    b. Find the height of the rectangle shown. Interpret your answer.



Answer

To approximate the area of region A, split the region into 10 rectangles. One has been shown as an example.

a. What is the width of each rectangle?

$$\frac{1-0}{10} = \frac{1}{10} = \frac{\Delta x}{10}$$

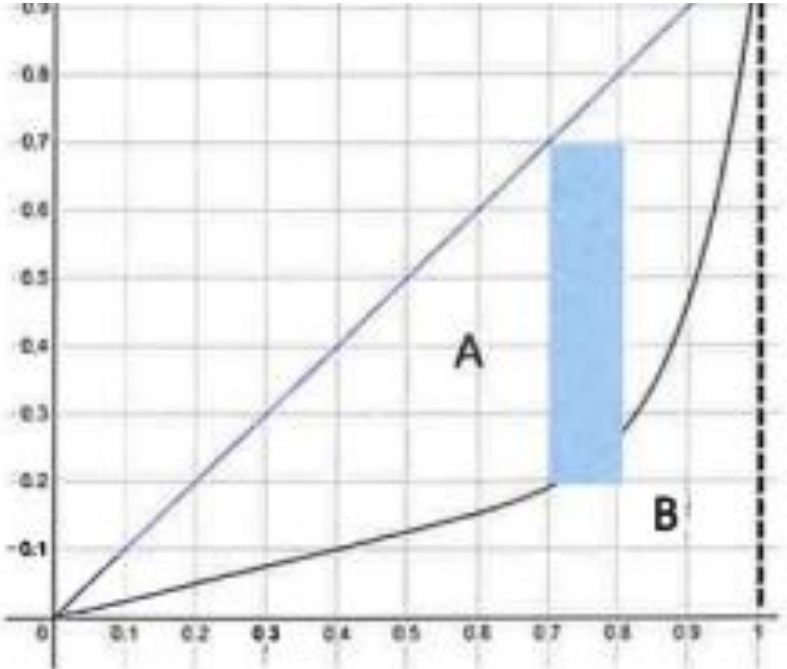
b. Find the height of the rectangle shown. Interpret your answer.

$$0.7 - L(0.7) = 0.7 - 0.2648 = 0.435$$

There is a 43% gap between what the poorest 70% earn and what they would earn in perfect equality.

Writing an integral

Write an integral expression that would give the area of region A using infinitely many rectangles.

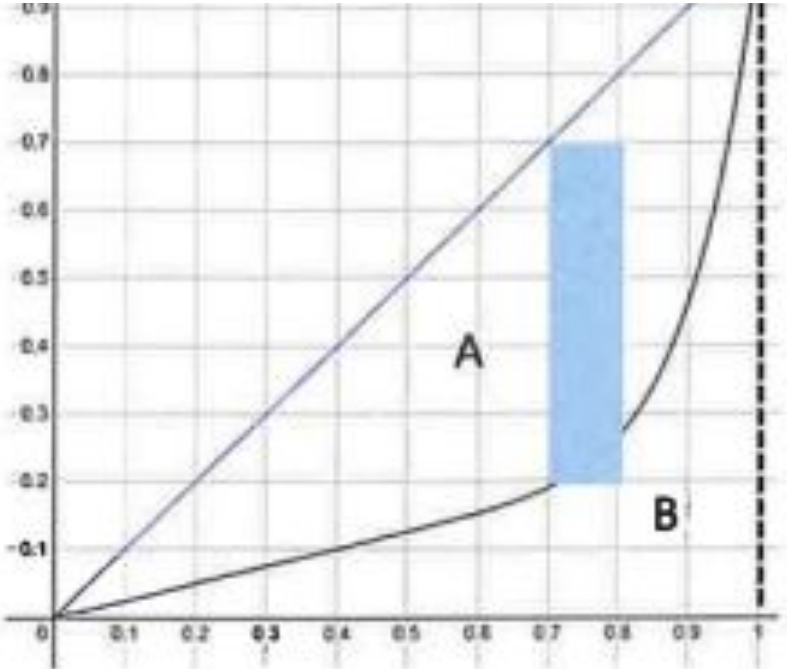


Writing an integral- Answer

Write an integral expression that would give the area of region A using infinitely many rectangles.

$$\int_0^1 (x - L(x))\,dx$$

height width



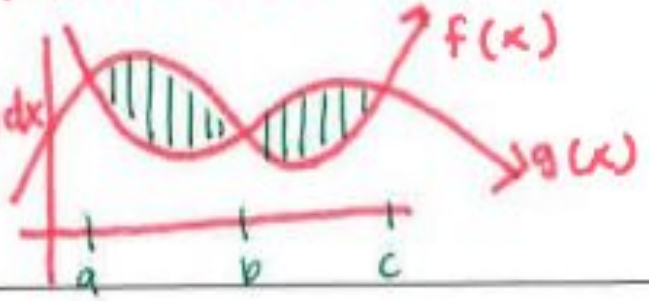
Area Between Curves

Watch this video for an example of how to calculate area between curves.

Important Ideas:

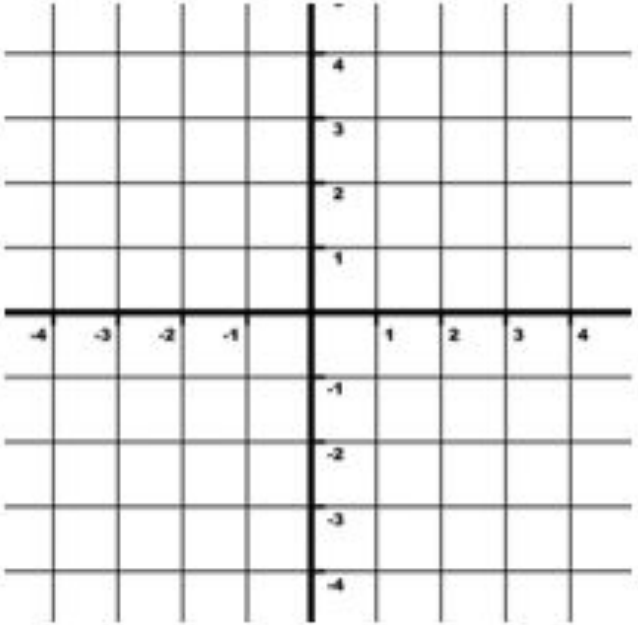
- Area between $f(x)$ and $g(x)$ on $[a,b]$ when $f(x) \geq g(x)$ is given by $A = \int_a^b (\underbrace{f(x)}_{\text{upper curve}} - \underbrace{g(x)}_{\text{lower curve}})\,dx$
- The region must be bounded.
- Area is ALWAYS positive.
- Sometimes the upper + lower functions switch!

$$A = \int_a^b (f(x)-g(x))\,dx + \int_b^c (g(x)-f(x))\,dx$$



Example 1

1. Find the area of the region enclosed by $y = x$ and $y = x^2 - 2$. Sketch a picture of the region.



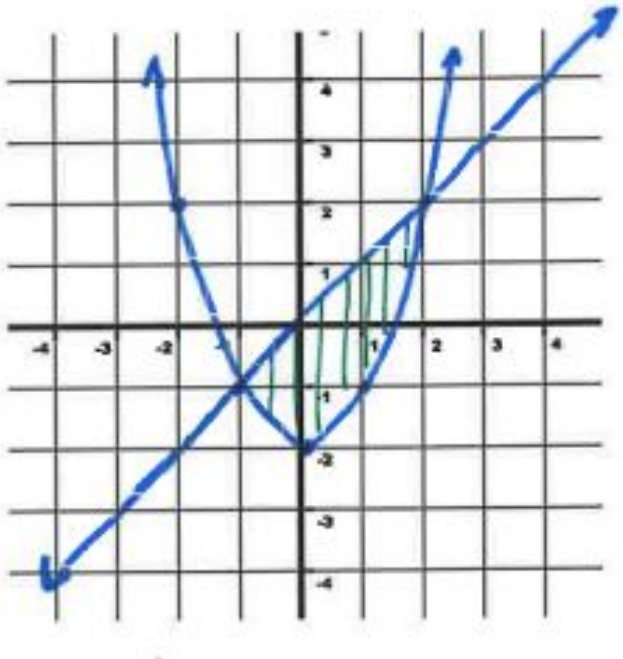
Example 1 Answer

1. Find the area of the region enclosed by $y = x$ and $y = x^2 - 2$. Sketch a picture of the region.

$$\int_{-1}^{2} (x - (x^2 - 2))\, dx$$

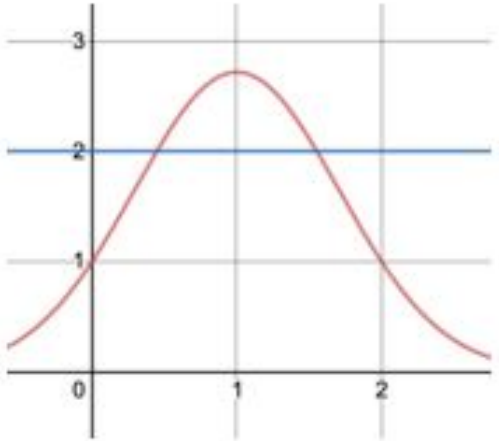
$$= \frac{x^2}{2} - \frac{x^3}{3} + 2x \Big]_{-1}^{2}$$

$$= \left(2 - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right) = 4.5$$



Example 2

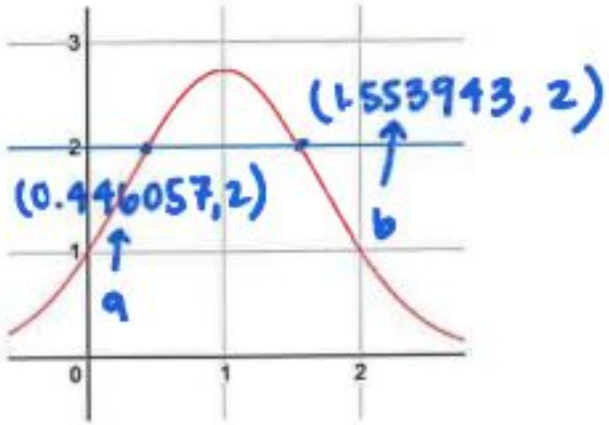
2. Let R be the region enclosed by $y = e^{2x-x^2}$ and $y = 2$. Find the area of R.



Example 2 Answer

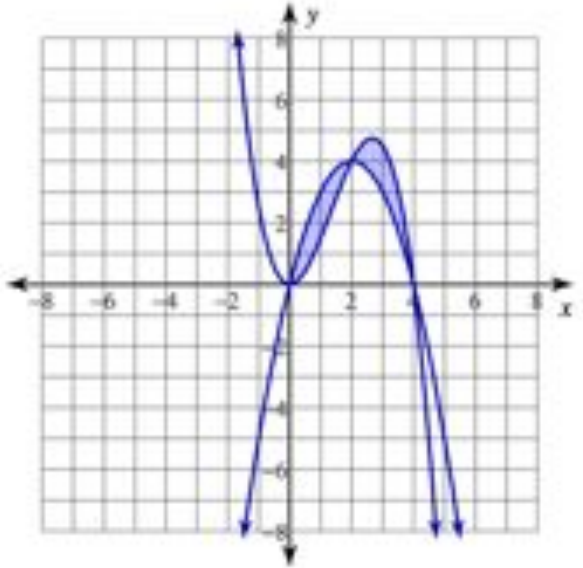
2. Let R be the region enclosed by $y = e^{2x-x^2}$ and $y = 2$. Find the area of R.

$$\int_a^b \left(e^{2x-x^2} - 2\right) dx = 0.514$$



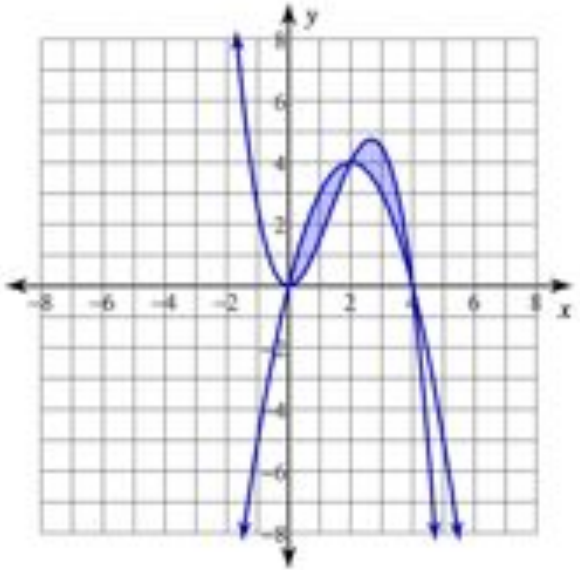
Example 3

3. Find the area of the region enclosed by the curves $y = -\frac{x^3}{2} + 2x^2$ and $y = 4x - x^2$.



Example 3- Answer

3. Find the area of the region enclosed by the curves $y = -\frac{x^3}{2} + 2x^2$ and $y = 4x - x^2$.



$$\int_0^2 \left[(4x - x^2) - \left(\frac{-x^3}{2} + 2x^2\right)\right] dx + \int_2^4 \left[\left(\frac{-x^3}{2} + 2x^2\right) - (4x - x^2)\right] dx$$

$$= 2 + 2 = 4$$

Practice

Extra Practice with Answers

Book Practice

Pg. 452: 6, 10, 22, 24, 42, 48

This lesson adapted with resources from Calc Medic