Inspiring Greatness

INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

AP Calculus AB

April 21, 2020

Lesson: April 21, 2020

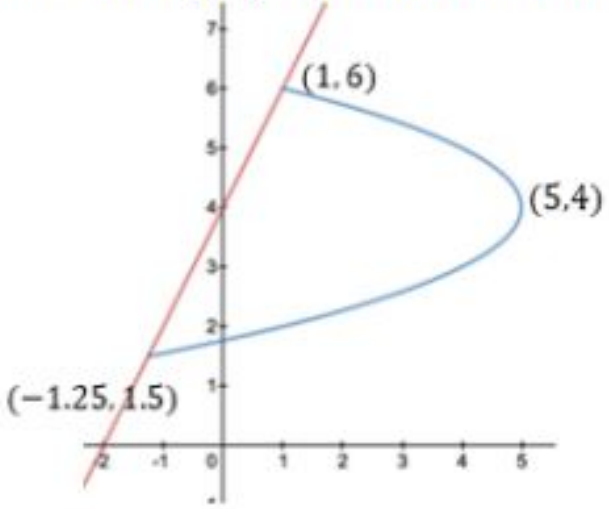
Objective/Learning Target:

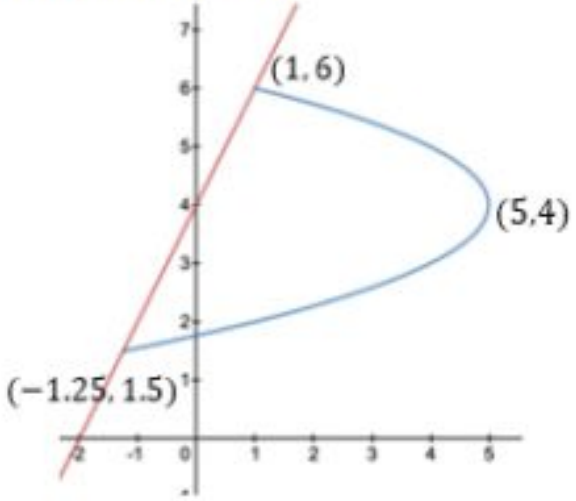
- I can calculate areas between curves expressed as functions of y
- I can find intersections of curves to determine the limits of integration
- I can use a sum of multiple definite integrals when curves intersect at more than two places

Review: Review the lesson from Monday, April 20
Watch: Area Between Curves in terms of y

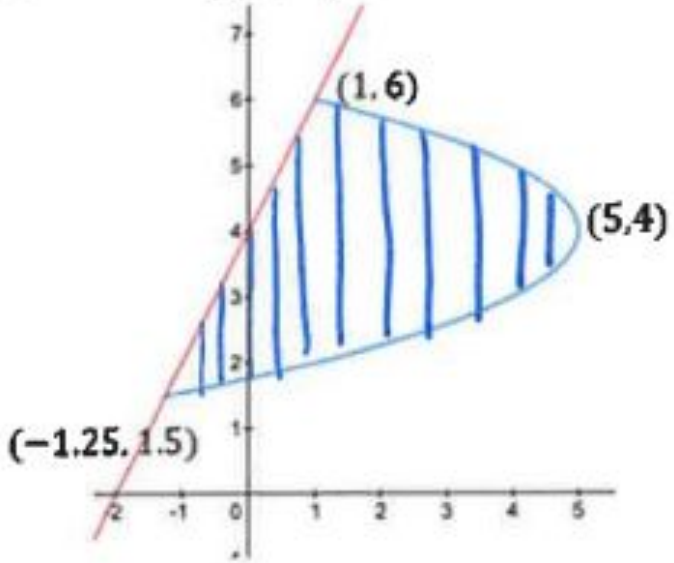
Builders use various methods for laying the planks on a deck. There are aesthetic and practical considerations to take into account when deciding on the decking pattern (diagonal, horizontal, herringbone, vertical). Today we will compare horizontal and vertical planking methods for decking a region.

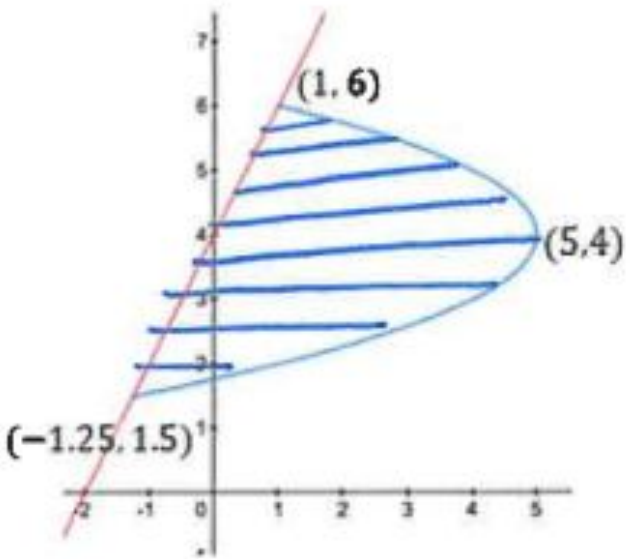
1. Imagine you are building a deck on the region shown below, enclosed by the curves $y = 2x + 4$ and $x - 5 = -(y - 4)^2$. On the first graph, sketch what it would look like to use vertical planks. On the second graph, sketch what it would look like to use horizontal planks.

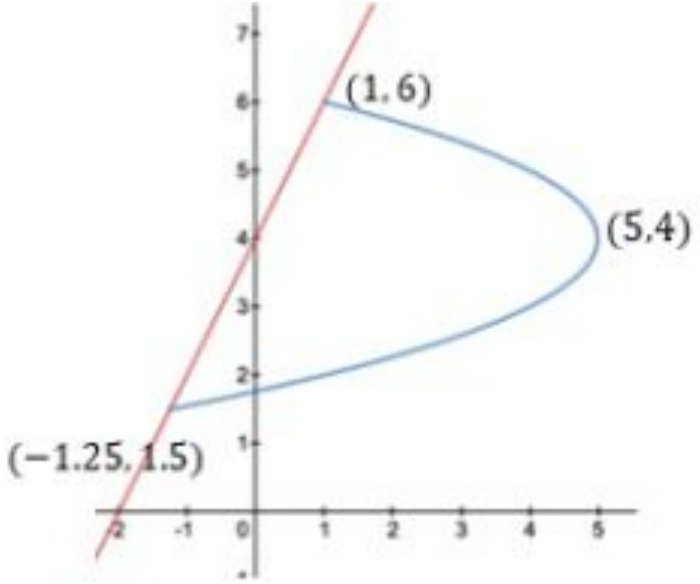


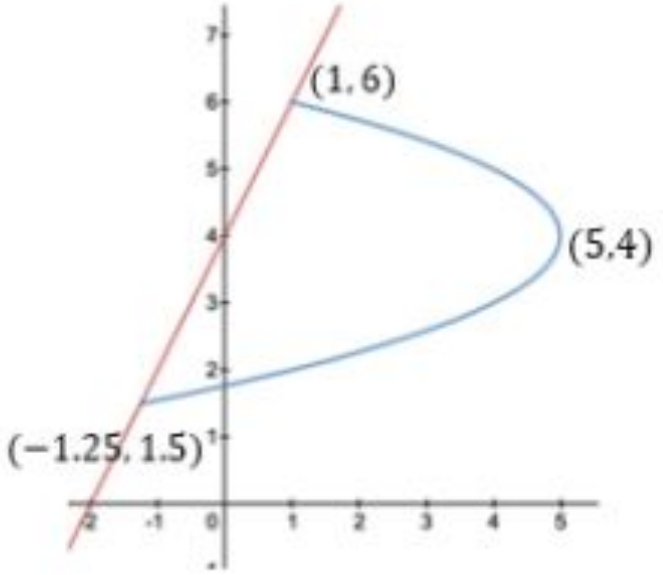


1. Imagine you are building a deck on the region shown below, enclosed by the curves $y = 2x + 4$ and $x - 5 = -(y - 4)^2$. On the first graph, sketch what it would look like to use vertical planks. On the second graph, sketch what it would look like to use horizontal planks.









2. When using vertical planks, at what x-value do the planks start? At what x-value do the planks end?

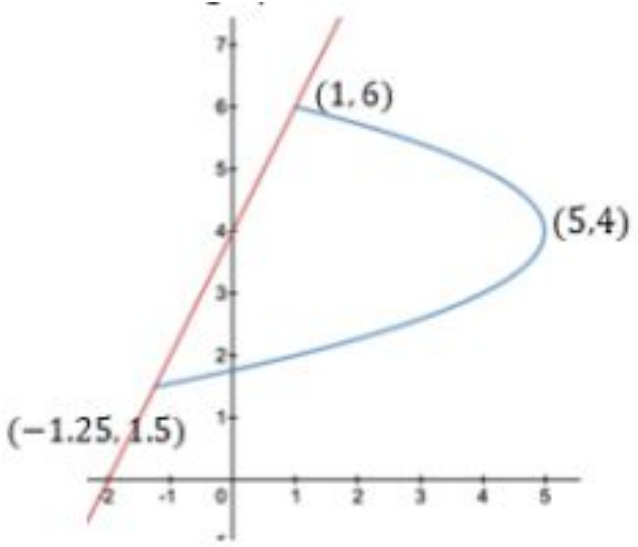
3. When using horizontal planks, at what y-value do the planks start? At what y-value do the planks end?

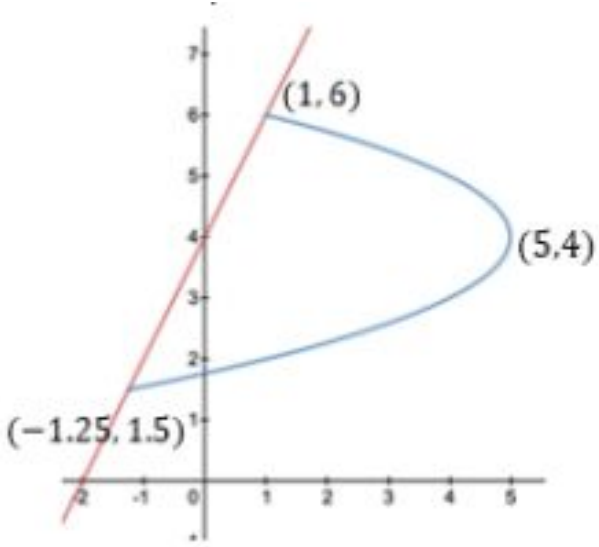
2. When using vertical planks, at what x-value do the planks start? At what x-value do the planks end?

$x = -1.25$ $x = 5$

3. When using horizontal planks, at what y-value do the planks start? At what y-value do the planks end?

$y = 1.5$ $y = 6$





4. If there are 20 planks used, how wide would each plank be if the planks are laid vertically? How wide would each plank be if the planks are laid horizontally?

5. Which method of laying the planks would make it easier to determine the length of each plank? Why?

4. If there are 20 planks used, how wide would each plank be if the planks are laid vertically? How wide would each plank be if the planks are laid horizontally?

$\frac{\Delta x}{20}$ $\frac{5-(-1.25)}{20} = 0.3125$ vertical

$\frac{6-1.5}{20} = 0.225$ $\frac{\Delta y}{20}$ horizontal

5. Which method of laying the planks would make it easier to determine the length of each plank? Why? Horizontal planks because the length of each plank is the difference between the x-values of the 2 curves. When using vertical planks, the upper & lower curve change.

6. Write an expression that would give the length of a horizontal plank at a given y-value.

Hint:

Solve the given equations for x.

Length = $x_2 - x_1$
= $f(y) - g(y)$

Answer

$x = -(y-4)^2 + 5$ $\qquad$ $x = \frac{1}{2}(y-4)$

$$\left(5-(y-4)^2\right) - \left(\tfrac{1}{2}(y-4)\right) = \text{length of plank}$$

7. What is the exact area of the deck?

Hint:

Use a definite integral.

Use your answer from #3 and #6.

7. What is the exact area of the deck?

y-values

$$\int_{1.5}^{6} \left(\left(5-(y-4)^2\right) - \left(\tfrac{1}{2}(y-4)\right) \right) dy = 15.1875$$

length of plank

width of plank

Important Ideas:

If the upper curve requires 2 or more definitions, consider using a right curve and a left curve (horizontal rectangles)

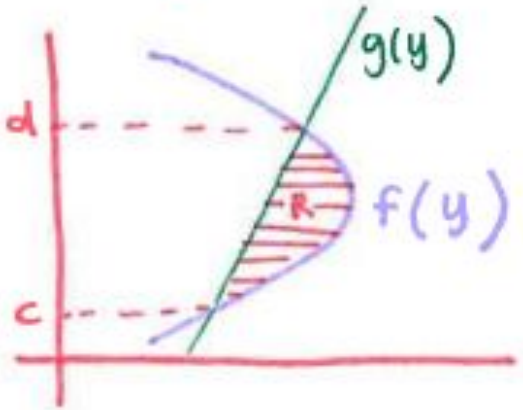
$$\text{Area of Region } R = \int_{c}^{d} \left(f(y) - g(y)\right) dy$$

y-values

$f(y)$: right curve

$g(y)$: left curve

dy: width

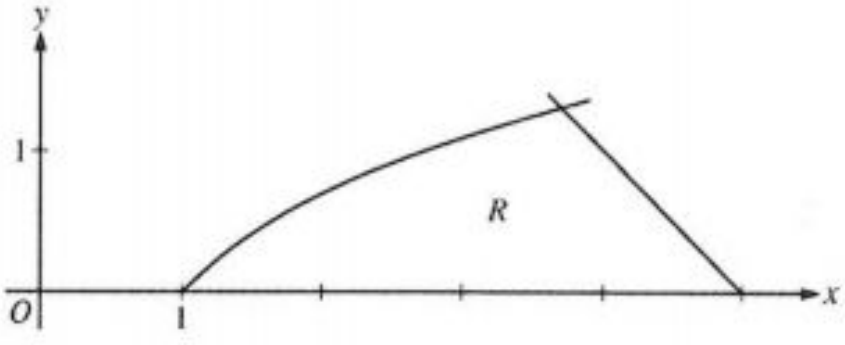


Examples

1. Let R be the region in the first quadrant bounded by the curves $y = \ln x$, $y = 5 - x$, and the x-axis, as shown below.

   a. Find the point of intersection of the two curves.

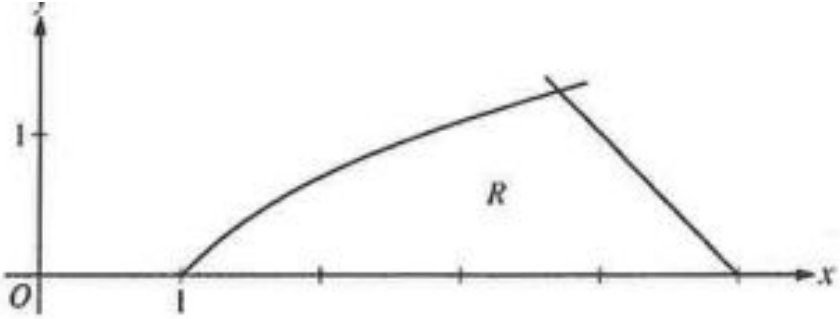
   b. Write an integral expression in terms of x that would give the area of region R.



Example Answer

a. Find the point of intersection of the two curves.

(3.6934414, 1.3065586)



b. Write an integral expression in terms of x that would give the area of region R.

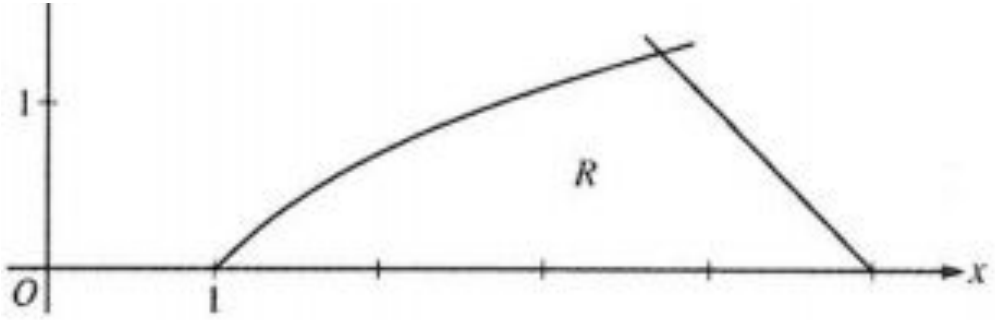
$$\int_{1}^{3.6934414} \ln x \, dx + \int_{3.6934414}^{5} (5 - x) \, dx$$

Example

c. Write an integral expression in terms of y that would give the area of region R.

Rewrite the equation to solve for x.

d. Find the area of region R.



Example Answer

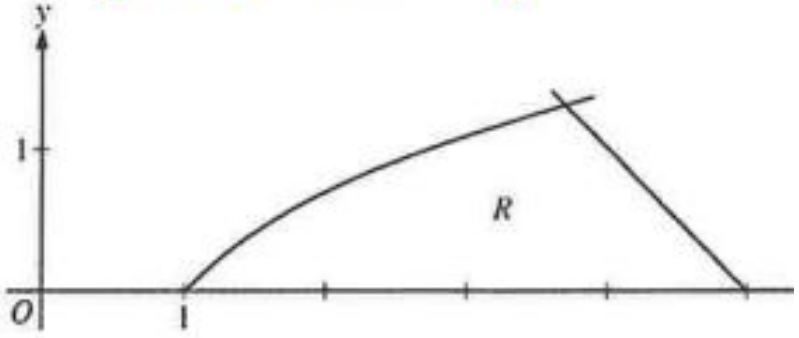
c. Write an integral expression in terms of y that would give the area of region R.

$$\int_0^{1.3065586} \left((5-y) - e^y \right) dy$$

d. Find the area of region R.

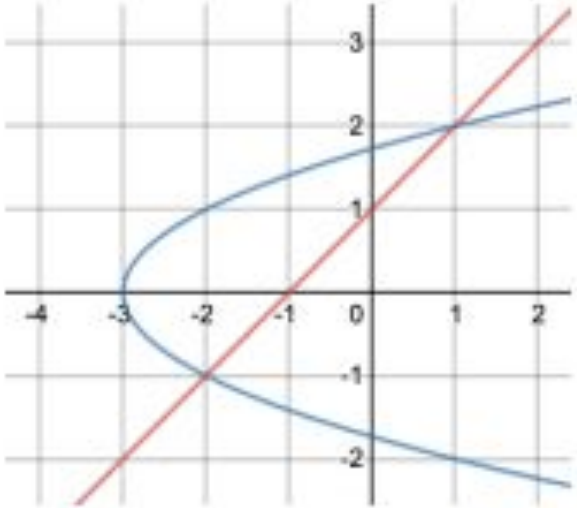
2.986

$e^y = x \qquad x = 5 - y$



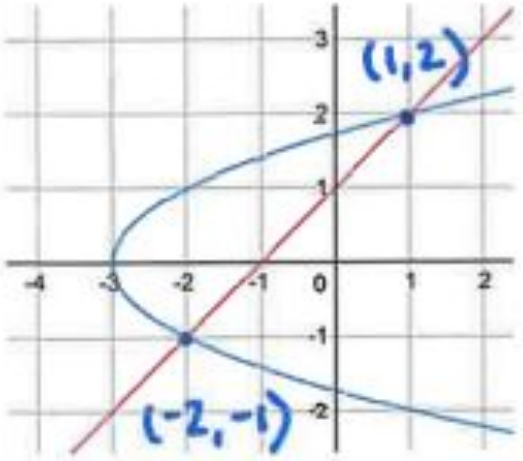
Example 2

2. Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$.



Example 2 Answer

2. Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$.



$x = y + 1$

$$\int_{-1}^{2} \left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right) \right] dy = 12$$

Practice

Practice with Answers

Textbook Problems:

Pg. 452: 14, 27, 30

Lesson lesson adapted from resources found on Calc Medic