Math Virtual Learning

Calculus AB

April 27, 2020

Lesson: Monday, April 27, 2020

Objective/Learning Target:

DAY 1 LIMIT REVIEW

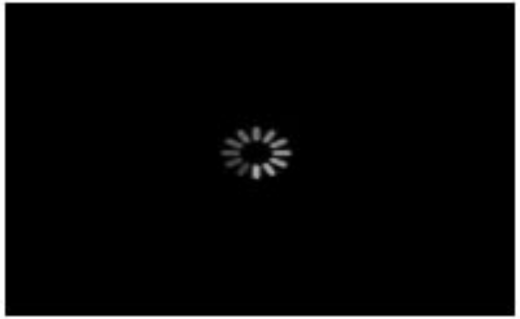
Estimate a limit using a numerical or graphical approach.

Learn different ways that a limit can fail to exist.

Study and use a formal definition of limit.

Getting Started

Steve Nash has the second highest lifetime free throw percent at 90.43%. You decide to study his free throw in greater detail to improve your own shot, but unfortunately the video freezes. How accurately can you guess the height of the ball after 5 milliseconds?



1. What information would you want to know to make your prediction?

Answer

Steve Nash has the second highest lifetime free throw percent at 90.43%. You decide to study his free throw in greater detail to improve your own shot, but unfortunately the video freezes. How accurately can you guess the height of the ball at time after 5 milliseconds?

1. What information would you want to know to make your prediction?

height of ball right before video froze

height of ball right after video froze

Introduction

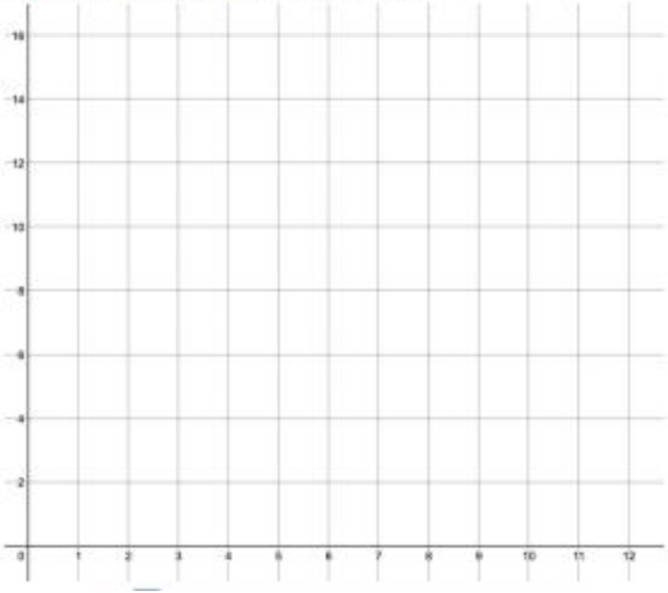
Suppose you knew that the height of the ball (in feet), *t* milliseconds after release could be modeled by the function $h(t) = -0.16t^2 + 2.4t + 7$.

a. Using a calculator, graph the function. Label your axes.

b. Trace the graph as *t* increases toward 5. What y-value is being approached?

c. Trace the graph as *t* decreases towards 5. What y-value is being approached?

d. Without the graph, how else could we have found the height of the ball as *t* gets closer and closer to 5?



Introduction- Answer

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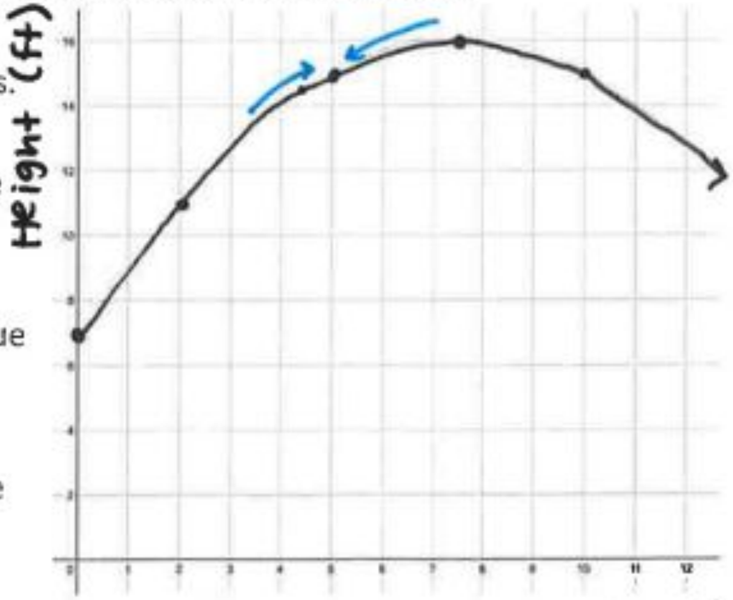


b. Trace the graph as t increases toward 5. What y-value is being approached? 15

c. Trace the graph as t decreases towards 5. What y-value is being approached? 15

d. Without the graph, how else could we have found the height of the ball as t gets closer and closer to 5?

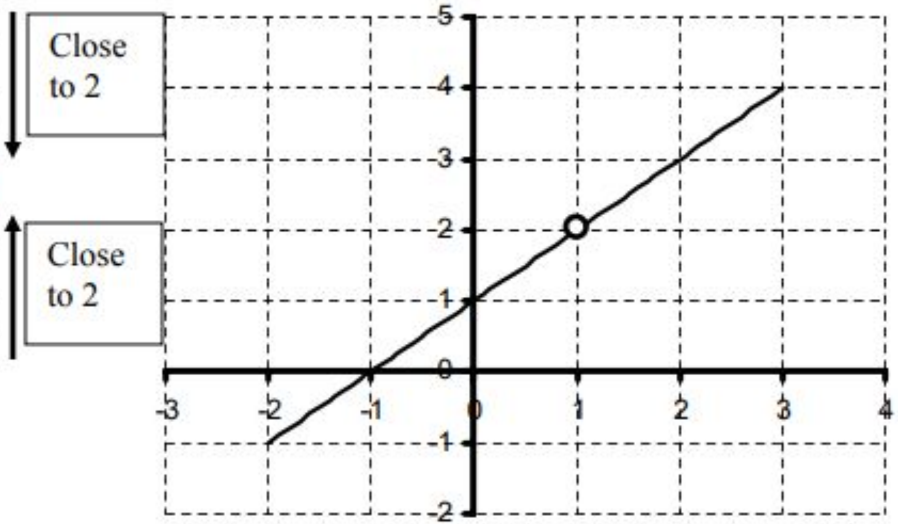
Plug t=5 into equation!

Example

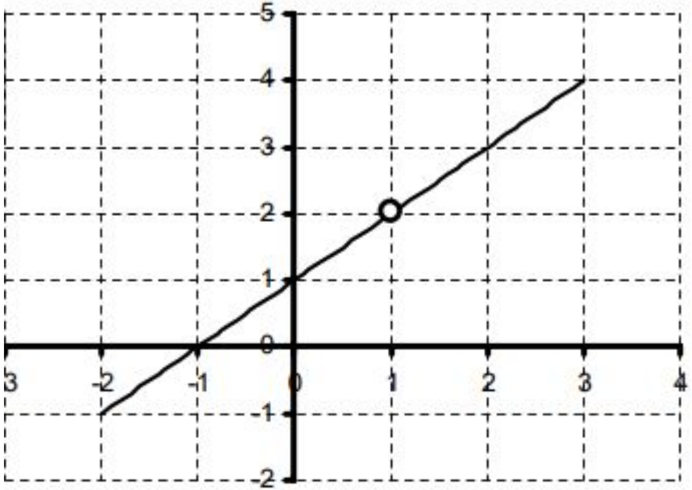
Consider the function $f(x) = \frac{x^2 - 1}{x - 1}$. What happens at $x = 1$?

We certainly can't find a function value there because $f(1)$ is undefined so the best we can do is to see what happens **near** the point $x = 1$. To do this we will graph the function. Since the function isn't easy to graph as it is written we will simply plot a few points:

x	$f(x)$
-1	0
0	1
.9	1.9
.99	1.99
1	undefined
1.01	2.01
1.1	2.1
2	3
3	4



There is a hole at $x = 1$ which is consistent with the fact that the function is undefined there. We can put in numbers into the function as **close** to 1 as we would like but we can't use 1 itself. Just saying that the function is undefined there doesn't tell us much about the function. What we would like to be able to answer is the question:



Q: What happens to $f(x)$ near $x = 1$?

A: As x gets **close** to 1 the value of $f(x)$ gets **close** to 2. We can see this from the graph. We read x across the horizontal axis the values for $f(x)$ are on the vertical axis. The graph is approaching the y value 2 as x gets **close** to 1.
We could also see this from the chart of values. You will notice that the $f(x)$ values are getting **close** to 2 as x is getting **closer** to 1.

We can write this mathematically the following way:

$\boxed{\lim_{x \to 1} f(x) = 2}$. We read this as "the limit as x approaches 1 of $f(x)$ is 2". There are the possible outcomes to a limit:

1. the limit doesn't exist
2. the limit exists and is a number
3. the limit exists and is $\pm\infty$

There are also different ways of finding a limit. We have seen two ways of finding the limit:

1. We tried numbers close to $x = 1$ and we checked what happened.
2. We looked at the graph and we saw what the function value was near $x = 1$.

Reading the limit off a graph is the **easiest** way to find the limit. Trying to create a table on numbers will work if the function behaves well. If it tends to change values very quickly this method may not be very accurate.

NOTE: The most important thing to remember when solving for limits is that we only care about what is happening to the function **NEAR** the point and NOT what is happening at the point.

Notation

$$\lim_{x \to c} f(x) = L$$

$$\lim_{x \to c-} f(x) \qquad \lim_{x \to c+} f(x)$$

- A limit is used to describe the behavior of a function near a point, but not at the point.

$$\lim_{x \to \infty} f(x) = L$$

Ex 1: $\lim_{x \to 2} \frac{x-2}{x^2-4}$. Once again this is asking the question, "What happens to the function as x gets close to 2". We need a table with values close to 2.

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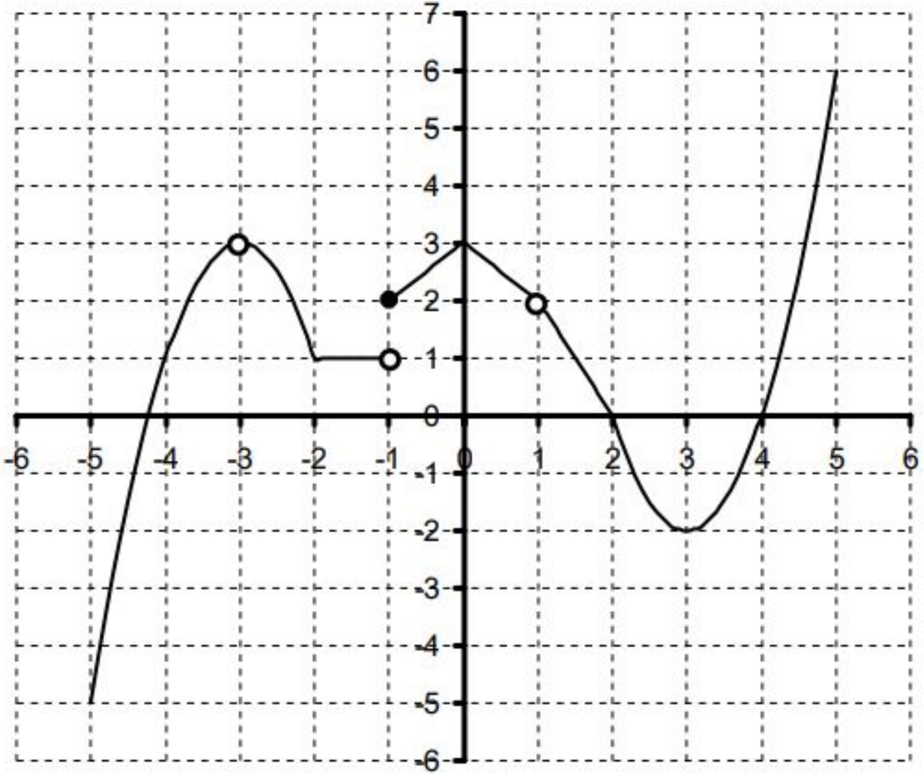
x	1.9	1.99	1.999	2	2.001	2.01	2.1
$\frac{x-2}{x^2-4}$	.25641	.250627	.25006	undefined	.249938	.249377	.243902

→ Close to 0.25

← Close to 0.25

You can tell that the values are getting close to 0.25 or 1/4.

Ex 3: Use the graph of $f(x)$ below to find the limits



Ex 2: $\lim_{x\to 4}\dfrac{\left(\dfrac{x}{x+1}\right)-\dfrac{4}{5}}{x-4}$

a) $\lim_{x\to -3} f(x) =$

b) $\lim_{x\to 0} f(x) =$

c) $\lim_{x\to -2} f(x) =$

d) $\lim_{x\to 1} f(x) =$

e) $\lim_{x\to -1} f(x) =$

Ex 2: $\lim_{x \to 4} \frac{\left(\frac{x}{x+1}\right) - \frac{4}{5}}{x-4}$

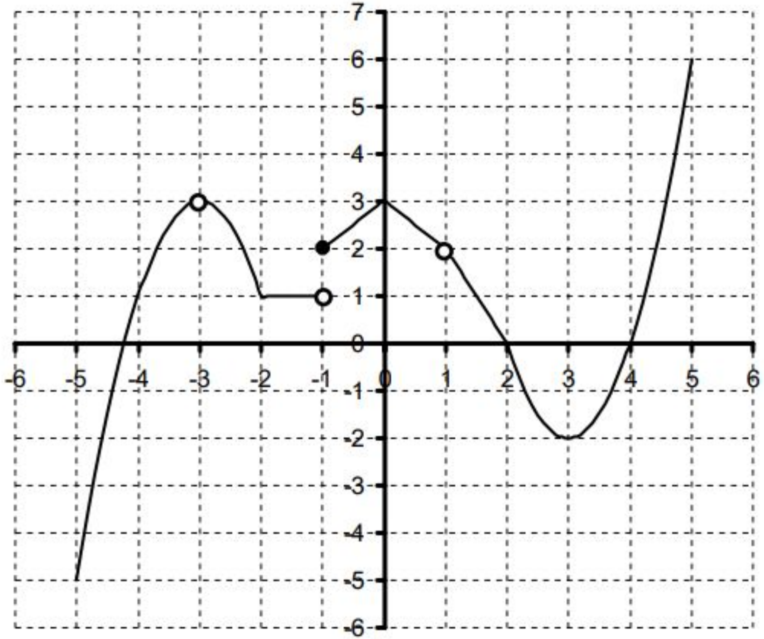
x	3.9	3.99	3.999	4	4.001	4.01	4.1
$\frac{\left(\frac{x}{x+1}\right) - \frac{4}{5}}{x-4}$	.040816	.04008	.04000	undefined	.03999	.03992	.03921

→ Close to 0.04

← Close to 0.04

Here the values are approaching 0.04

Ex 3: Use the graph of $f(x)$ below to find the limits



a) $\lim_{x \to -3} f(x) = \boxed{3}$

b) $\lim_{x \to 0} f(x) = \boxed{3}$

c) $\lim_{x \to -2} f(x) = \boxed{1}$

d) $\lim_{x \to 1} f(x) = \boxed{2}$

e) $\lim_{x \to -1} f(x) =$ Does Not Exist because the function does not come to one place at $x = 1$. From one side the function values are approaching 1 and from the other side the values are approaching 2.

Ex 4: $\lim_{x \to 0} \frac{|x|}{x}$

We will do this problem 2 ways. By a chart and by a graph.

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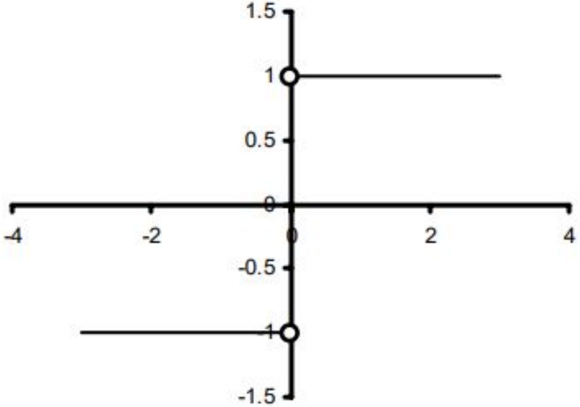
x	-3	-2	-1	0	1	2	3
$\frac{|x|}{x}$	-1	-1	-1	undefined	1	1	1

→ Close to -1 ← Close to 1



From either of these methods we can see that the function does not approach one unique value. From one side we are at -1 and from the other side we are at +1. Since these are not the same the limit does not exist.

Practice

Online Worksheet with Answers Click on the video camera in the bottom corner of each problem for the answer.

Textbook Suggested Problems: Pg. 55: 1, 5, 9-21 all, 23, 25, 63, 65-67