Math Virtual Learning

Calculus AB

Review of Riemann Sums

May 11, 2020

Lesson: Monday, May 11, 2020

Objective/Learning Target:

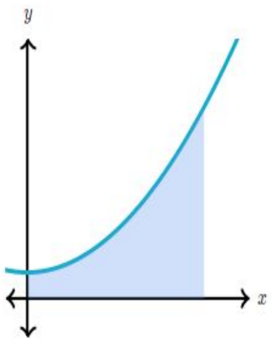
Understand the definition of a Riemann Sum

Approximate the area of a plane region

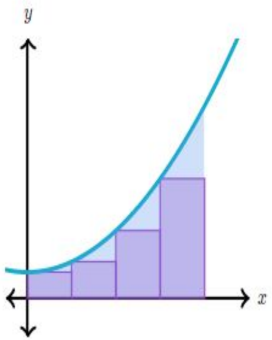
Introduction: Intro to Riemann Sums

Over and Under- estimates

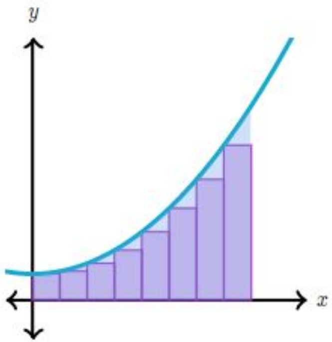
Suppose we want to find the area under this curve:

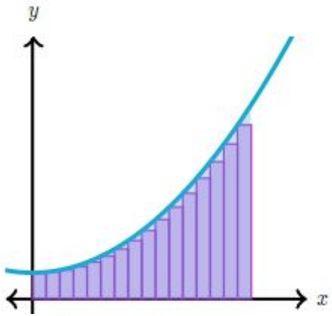


We may struggle to find the exact area, but we can approximate it using rectangles:



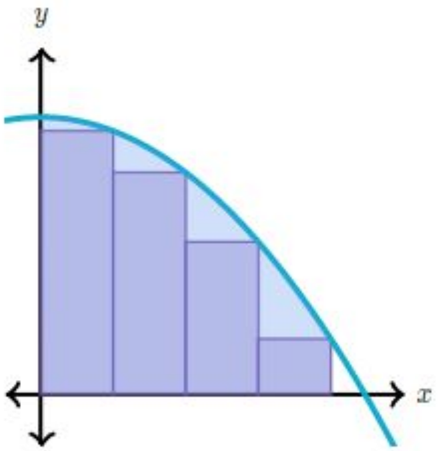
And our approximation gets better if we use more rectangles:





These sorts of approximations are called **Riemann sums**, and they're a foundational tool for integral calculus. Our goal, for now, is to focus on understanding two types of Riemann sums: left Riemann sums, and right Riemann sums.

What kind of Riemann sum is described by the diagram?



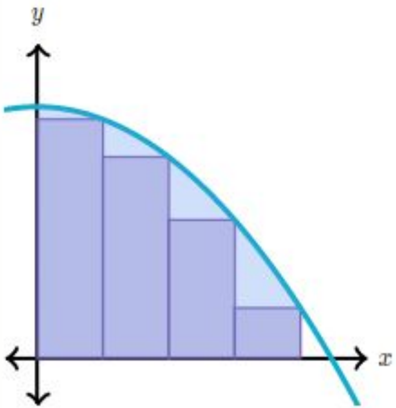
Choose 1 answer:

(A) Left Riemann sum

(B) Right Riemann sum

PROBLEM 1

What kind of Riemann sum is described by the diagram?



Choose 1 answer:

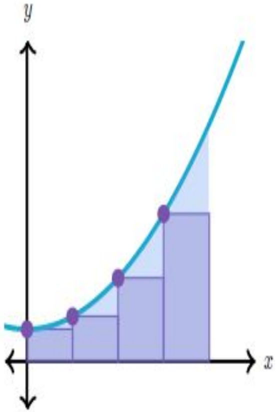
(A) Left Riemann sum

CORRECT (SELECTED)

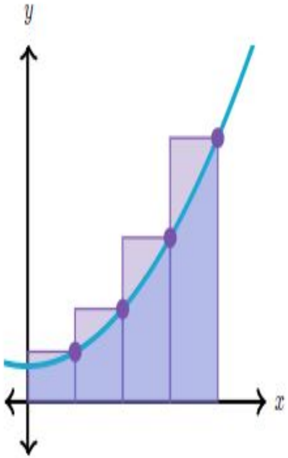
(B) Right Riemann sum

Left and right Riemann sums

To make a Riemann sum, we must choose how we're going to make our rectangles. One possible choice is to make our rectangles touch the curve with their top-left corners. This is called a **left Riemann sum**.



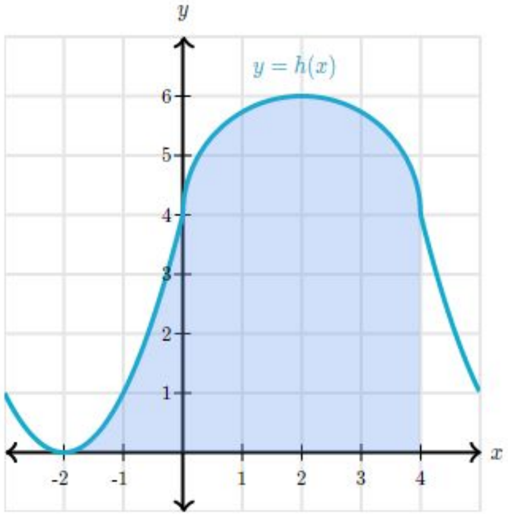
Another choice is to make our rectangles touch the curve with their top-right corners. This is a **right Riemann sum**.



Neither choice is strictly better than the other.

Try it.

Approximate the area between $y = h(x)$ and the x-axis from $x = -2$ to $x = 4$ using a *right Riemann sum* with *three* equal subdivisions.



Choose 1 answer:

(A) 20 units^2

(B) 26.5 units^2

(C) 28 units^2

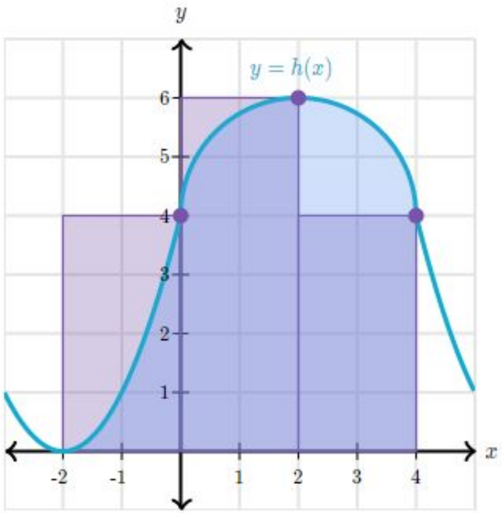
Answer

CORRECT (SELECTED)

28 units^2

Check

Hide explanation



	First rectangle	Second rectangle	Third rectangle
Width	2	2	2
Height	4	6	4
Area	$2 \cdot 4 = 8$	$2 \cdot 6 = 12$	$2 \cdot 4 = 8$

Then, after finding the individual areas of the rectangles, we'd add them up to get 28 units^2.

Other Ways of Calculating Area

[Area from a Table]

[Midpoint Sum]

[Trapezoidal Sum]

Practice

1. Consider the region enclosed between the x-axis and the curve $y = e^x$.
   a. Use a left Riemann sum approximation with 5 equal subintervals to approximate the area of the region between $x = -1$ and $x = 4$. Show your work.

   b. Use a midpoint Riemann sum approximation with 5 equal subintervals to approximate the same region. Show your work.

2. The rate at which water flows out of a pipe in gallons per hour is given by $R(t)$. Selected values of $R(t)$ are shown in the table below.
   a. Use a right Riemann sum approximation with 4 equal subintervals to approximate the area underneath $R(t)$ from $t = 0$ to $t = 24$.

   b. What does this area represent?

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

Practice Answers

1. Consider the region enclosed between the x-axis and the curve $y = e^x$.
   a. Use a left Riemann sum approximation with 5 equal subintervals to approximate the area of the region between $x = -1$ and $x = 4$. Show your work.

$1(e^{-1}) + 1(e^{0}) + 1(e^{1}) + 1(e^{2}) + 1(e^{3})$

$= 31.561$

   b. Use a midpoint Riemann sum approximation with 5 equal subintervals to approximate the same region. Show your work.

$1(e^{-0.5}) + 1(e^{0.5}) + 1(e^{1.5}) + 1(e^{2.5}) + 1(e^{3.5})$

$= 52.035$

Practice Answers

2. The rate at which water flows out of a pipe in gallons per hour is given by $R(t)$. Selected values of $R(t)$ are shown in the table below.
   a. Use a right Riemann sum approximation with 4 equal subintervals to approximate the area underneath $R(t)$ from $t = 0$ to $t = 24$.

$6(10.8) + 6(11.4) + 6(10.7) + 6(9.6) = 255$

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

   b. What does this area represent?

The amount of water that flows out of the pipe between $t=0$ and $t=24$ hours

Practice

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

Practice

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

Answers

(b) $\frac{3}{2}(20 + 2(31) + 2(28) + 2(24) + 2(22) + 21) = 376.5$

Average temperature $\approx \frac{1}{15}(376.5) = 25.1\,^{\circ}\mathrm{C}$

Answers

(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$

$= 19.3$ ft

$\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.

(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.

Units of ft^3/min in part (b) and ft in part (c)

Extra Practice

[Riemann Sums with Tables]

[Approximating Area Under a Curve]

Textbook Suggested Problems:

Section 4.3 pg. 278-279: 15, 19, 27, 29, 35, 37, 43, 45, 47