Math Virtual Learning

Calculus AB

Review of Derivatives- Product and Quotient Rules

May 4, 2020

Lesson: Monday, May 4, 2020

Objective/Learning Target:
Lesson 1 Derivatives Review
Find the derivative using Power Rule
Find the derivative using Product and Quotient Rule
Find the derivative of all trig functions

Introduction Videos

Power Rule

Power Rule with Rewriting the Function

Derivative of Sine and Cosine

Product Rule

Quotient Rule

Tan and Cot Derivative

Sec and Csc Derivative

Power Rule Examples

Suppose $f(x) = x^{2/3} + 4x^{-6} - 3x^{-1/5}$. Find $f'(x)$.

$$f'(x) = \frac{2}{3}x^{\frac{2}{3}-1} + 4(-6)x^{-6-1} - 3\left(-\frac{1}{5}\right)x^{-\frac{1}{5}-1}$$
$$= \frac{2}{3}x^{\frac{2}{3}-\frac{3}{3}} - 24x^{-7} + \frac{3}{5}x^{-\frac{1}{5}-\frac{5}{5}}$$
$$= \frac{2}{3}x^{-1/3} - 24x^{-7} + \frac{3}{5}x^{-6/5}$$

Answer

$f'(x) = \frac{2}{3}x^{-1/3} - 24x^{-7} + \frac{3}{5}x^{-6/5}$ when $f(x) = x^{2/3} + 4x^{-6} - 3x^{-1/5}$

Suppose $f(x) = \sqrt[4]{x} + \frac{6}{\sqrt{x}}$. Find $f'(x)$.

Step 1

Rewrite the function so each term is a power function (i.e., has the form ax^n).

$$f(x) = \sqrt[4]{x} + \frac{6}{\sqrt{x}}$$
$$= x^{1/4} + \frac{6}{x^{1/2}}$$
$$= x^{1/4} + 6x^{-1/2}$$

Step 2

Use the power rule for derivatives to differentiate each term.

$$f(x) = x^{1/4} + 6x^{-1/2}$$
$$= \frac{1}{4}x^{\frac{1}{4}-1} + 6\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$$
$$= \frac{1}{4}x^{\frac{1}{4}-\frac{4}{4}} - 3x^{-\frac{1}{2}-\frac{2}{2}}$$
$$= \frac{1}{4}x^{-3/4} - 3x^{-3/2}$$

Practice- Simplify, if necessary.

$f(x) = \frac{8}{\sqrt{x}} - 3x$

$f(x) = x(x+1)$

$f(x) = \frac{x^2-1}{x}$

$f(x) = \frac{7x+3x^2}{5\sqrt{x}}$

Practice- Answers

$f(x) = \frac{8}{\sqrt{x}} - 3x$

$-4x^{-\frac{3}{2}} - 3$

$f(x) = x(x+1)$

$2x+1$

$f(x) = \frac{x^2-1}{x}$

$1 + x^{-2}$

$f(x) = \frac{7x+3x^2}{5\sqrt{x}}$

$\frac{7}{10}x^{-\frac{1}{2}} + \frac{9}{10}x^{\frac{1}{2}}$

Product and Quotient Rule Formulas

Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Product Rule Example

If we have a product like

$$y = (2x^2 + 6x)(2x^3 + 5x^2)$$

we can find the derivative without multiplying out the expression on the right.

We can then use the PRODUCT RULE:

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

We first find: $\frac{dv}{dx} = 6x^2 + 10x$ and $\frac{du}{dx} = 4x + 6$

Then we can write:

$$\begin{aligned}\frac{d(uv)}{dx} &= u\frac{dv}{dx} + v\frac{du}{dx} \\ &= (2x^2 + 6x)(6x^2 + 10x) + (2x^3 + 5x^2)(4x + 6) \\ &= 20x^4 + 88x^3 + 90x^2\end{aligned}$$

You can stop here
(especially on the AP test)

Quotient Rule Example

We wish to find the derivative of the expression:

$$y = \frac{2x^3}{4-x}$$

We recognise that it is in the form: $y = \frac{u}{v}$.

We can use the substitutions:

$$u = 2x^3 \text{ and } v = 4 - x$$

Using the **quotient rule**, we first need to find:

$$\frac{du}{dx} = 6x^2$$

and

$$\frac{dv}{dx} = -1$$

Then

$$\begin{aligned}
\frac{d\left(\frac{u}{v}\right)}{dx} &= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \\
&= \frac{(4-x)\left(6x^2\right) - \left(2x^3\right)(-1)}{(4-x)^2} \\
&= \frac{24x^2 - 6x^3 + 2x^3}{(4-x)^2} \\
&= \frac{24x^2 - 4x^3}{(4-x)^2}
\end{aligned}$$

Practice

$$y = \left(1 + \frac{1}{x^5}\right)(3x^4 - 2)$$

$$y = \frac{3x^5 - 5x^4 - x^2}{4x^5 - 4}$$

Practice- Answers

$$y = \left(1 + \frac{1}{x^5}\right)(3x^4 - 2)$$

$$y = \frac{3x^5 - 5x^4 - x^2}{4x^5 - 4}$$

$$y = \left(1 + \frac{1}{x^5}\right)(3x^4 - 2)$$

$$\frac{dy}{dx} = \left(1 + x^{-5}\right) \cdot 12x^3 + \left(3x^4 - 2\right) \cdot -5x^{-6}$$

$$= 12x^3 - \frac{3}{x^2} + \frac{10}{x^6}$$

$$y = \frac{3x^5 - 5x^4 - x^2}{4x^5 - 4}$$

$$\frac{dy}{dx} = \frac{(4x^5 - 4)(15x^4 - 20x^3 - 2x) - (3x^5 - 5x^4 - x^2) \cdot 20x^4}{(4x^5 - 4)^2}$$

$$= \frac{5x^8 + 3x^6 - 15x^4 + 20x^3 + 2x}{4x^{10} - 8x^5 + 4}$$

More Practice

Worksheet 1

Power, Constant and Sum Rules

Product Rule Worksheet

Quotient Rule Worksheet

Textbook Practice-

2.2 Pg. 115: 1, 3-31 by 4, 39, 43, 51, 55, 59, 61, 63, 67,-72, 83-89, 91, 93, 97-100, 103

2.3 Pg. 126: 1, 5, 7, 11, 15, 17, 19, 23-51 by 4, 61, 69, 73, 83, 87, 93-101 by 4, 103-111, 113, 129-134