Calculus AB
Lesson: Monday, April 6th

Learning Target:
Students will integrate Natural Logs

Review:
U-Substitution and Integration **(read)**
U-Substitution and Integration **(video)**

Practice:

1. Review DERIVATIVES of natural log functions with these practice problems and answers (Section 5.2).

$$\frac{d}{dx}\left[\ln|u|\right] = \frac{u'}{u}$$

$y = \ln\left(x^2+1\right)$

$u = x^2 + 1$

$du = 2x\ dx$

$y' = \frac{1}{u} \cdot \frac{d}{du}$

$y' = \frac{1}{x^2+1} \cdot 2x$

$y' = \frac{2x}{x^2+1}$

2. We often rewrite problems to be in the form of a polynomial. We know to use the integral format for natural log when rewriting and the exponent is equal to -1.

Represents the integral of a polynomial:

$$\int u^n\, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1.$$

$$\int \frac{1}{u}\, dx = \ln|u| + C \qquad n = -1$$

Notes and Examples

$$\int \frac{1}{x-5}\,dx$$

$u = x-5$

$du = dx$

$$\int \frac{1}{u}\,du$$

$= \ln|u| + C$

$= \ln|x-5| + C$

Try this one on your own… Show the work needed to get the answer.

$$\int \frac{20x^4}{4x^5+3}\,dx;\quad u = 4x^5+3$$

$\ln\left|4x^5+3\right| + C$

$$\int \frac{3-4x}{6+3x-2x^2}\,dx = \ln(6+3x-2x^2)+c$$

Long Division Examples

In order to evaluate some functions, we must rewrite them first. Long division is one way to rewrite a rational function.

$$\int \frac{x^2-3x+2}{x+1}\,dx =$$

$$
\begin{array}{r l}
 & x-4+\frac{6}{x+1} \\
x+1 \,\big) & x^2-3x+2 \\
 & -(x^2+x) \\
 & \quad -4x+2 \\
 & \quad -(-4x-4) \\
 & \qquad 6
\end{array}
$$

$$\int x-4\,dx+\int \frac{6}{x+1}\,dx$$

$$\frac{1}{2}x^2-4x+6\int\frac{dx}{x+1}$$

$$\frac{1}{2}x^2-4x+6\ln|x+1|$$

$$+C$$

Another Long Division Example

$$\int \frac{x^3 - 6x - 20}{x + 5}$$

$$x^2 - 5x + 19 - \frac{115}{x+5}$$

$$x+5 \overline{)\, x^3 + 0x^2 - 6x - 20}$$

$$-(x^3 + 5x^2)$$

$$-5x^2 - 6x$$

$$-(-5x^2 - 25x)$$

$$19x - 20$$

$$-(19x + 95)$$

$$-115$$

$$\int x^2 - 5x + 19\,dx - 115 \int \frac{dx}{x+5}$$

$$= \frac{1}{3}x^3 - \frac{5}{2}x^2 + 19x - 115 \ln|x+5| + C$$

Proving Integrals for other Trig Functions:

We already know the integral of sin and cos. We can now prove the integrals for the other trig functions.

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$-\int \frac{du}{u} = -\ln|u| + C$$

$$-\ln|\cos x| + C$$

$$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \ln|\sin x| + C$$

Proving Integrals for other Trig Functions:

The proof for sec and csc are a little tougher. Shown is the proof for the integral of secant. Cosecant is done in a similiar way.

$$\int \sec x\,dx = \int \sec x\left(\frac{\sec x+\tan x}{\sec x+\tan x}\right)dx$$

$$= \int \frac{\sec^2 x + \sec x\tan x}{\sec x + \tan x} = \int \frac{du}{u}$$

$u = \sec x + \tan x$

$du = \sec x\tan x + \sec^2 x\,dx$

$$= \ln\left|\sec x + \tan x\right| + C$$

$$\int \csc x\,dx = -\ln\left|\csc x + \cot x\right| + C$$

More Examples

$$\int \frac{1}{1+\sqrt{3x}}\,dx$$

$u = 1 + \sqrt{3x}$ $\quad (3x)^{1/2}$

$u - 1 = \sqrt{3x}$

$du = \frac{1}{2}(3x)^{-1/2} \cdot 3$

$du = \frac{3\,dx}{2\sqrt{3x}}$

$2\sqrt{3x}\,du = 3dx$

$\frac{2\sqrt{3x}}{3}\,du = dx$

$\frac{2}{3}(u-1)\,du = dx$

$\frac{2}{3}\int \frac{1}{u}\cdot(u-1)\,du$

$\frac{2}{3}\int \frac{u}{u} - \frac{1}{u}\,du$

$\frac{2}{3}\int 1 - \frac{1}{u}\,du = \frac{2}{3}\left[u - \ln|u|\right] + C$

$\frac{2}{3}\left[1+\sqrt{3x} - \ln\left|1+\sqrt{3x}\right|\right] + C$

$$\int \tan 5\theta\,d\theta$$

$u = 5\theta$

$du = 5\,d\theta$

$\frac{1}{5}du = d\theta$

$\frac{1}{5}\int \tan u\,du$

$= -\frac{1}{5}\ln|\cos 5\theta| + C$

Practice

Try these problems from the textbook:

p. 338 #5,11,17,25,33,51,63

[Extra Practice with Answers]