#### Calculus AB Lesson: Monday, April 6th

#### **Learning Target:**

Students will integrate Natural Logs

#### Review:

<u>U-Substitution and Integration</u> (read) <u>U-Substitution and Integration</u> (video)

#### **Practice:**

1. Review DERIVATIVES of natural log functions with these practice problems and answers (Section 5.2).

$$\frac{d}{dx} \left[ \ln |u| \right] = \frac{u}{u}$$

$$y = \ln(x^{2} + 1)$$

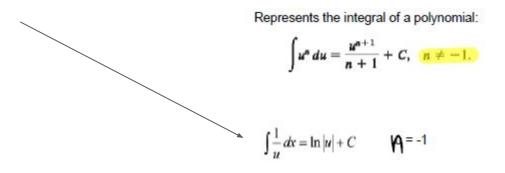
$$du = x^{2} + 1$$

$$du = 3x dx$$

$$y' = \frac{1}{x^{2} + 1} \cdot 2x$$

$$y' = \frac{2x}{x^{2} + 1}$$

We often rewrite problems to be in the form of a polynomial. We know to use the integral format for natural log when rewriting and the exponent is equal to -1.



## Notes and Examples

$$\int \frac{1}{x-5} dx$$

$$u = X-5$$

$$du = dx$$

$$\int \frac{1}{u} du$$

$$= |n|u| + C$$

$$= |n|x-5| + C$$

Try this one on your own... Show the work needed to get the answer.

$$\int \frac{20x^4}{4x^5 + 3} dx; \ u = 4x^5 + 3$$

$$\int \frac{3 - 4x}{6 + 3x - 2x^2} dx = \ln(6 + 3x - 2x^2) + c$$

$$\ln |4x^5 + 3| + C$$

## Long Division Examples

In order to evaluate some functions, we must rewrite them first. Long division is one way to rewrite a rational function.

$$\int \frac{x^{2}-3x+2}{x+1} dx = \int x-4 dx + \int \frac{6}{x+1} dx$$

$$x - 4 + \overline{x+1}$$

$$x + \int x^{2}-3x + 2$$

$$-(x^{2}+x) \int \frac{1}{2}x^{2}-4x + 6 \ln |x+1|$$

$$-(-4x-4) \int 6$$

## Another Long Division Example

$$\int \frac{x^3 - 6x - 20}{x + 5}$$

$$x^2 - 5x + 19 - \frac{115}{x + 5}$$

$$x + 5 \int x^3 + 0x^2 - 10x - 20$$

$$-(x^3 + 5x^2)$$

$$-5x^2 - 10x$$

$$-(-5x^3 - 25x)$$

$$19x - 20$$

$$-(19x + 95)$$

$$-115$$

$$\int x^2 - 5x + 19 \, dx - 105 \int \frac{dx}{x + 5}$$

$$= \frac{1}{3}x^3 - \frac{5}{2}x^2 + 19x - 115 \ln |x + 5| + c$$

## Proving Integrals for other Trig Functions:

We already know the integral of sin and cos. We can now prove the integrals for the other trig functions.

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx$$

$$\int u = \cos x$$

$$du = -\sin x dx$$

$$\int du = -\ln |u| + C$$

$$-\ln |\cos x| + C$$

$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx$$

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$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx$$

$$\int u = \sin x dx$$

$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx$$

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$$\int \frac{\cos x}{\sin x} dx$$

$$\int \frac{\sin x}{\sin x}$$

## Proving Integrals for other Trig Functions:

The proof for sec and csc are a little tougher. Shown is the proof for the integral of secant. Cosecant is done in a similar way.

$$\int \sec x dx = \int \sec x \left( \frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} = \int \frac{du}{u}$$

$$u = \sec x + \tan x = \ln \left| \sec x + \tan x \right| + c$$

$$du = \sec x + \tan x + \sec^2 x dx$$

$$\int \csc x dx = -|n| \csc x + \cot x + c$$

# More Examples

$$\int \frac{1}{1 + \sqrt{3x}} dx \qquad \int \tan 5\theta d\theta$$

$$U = 1 + \sqrt{3x} \qquad (3x)^{\frac{1}{2}} \qquad U = 5\theta$$

$$du = \frac{1}{2}(3x)^{\frac{1}{2}} \frac{3}{3} \qquad du = 5d\theta$$

$$du = \frac{3}{2} \frac{3}{3} \frac{3}{3} \qquad \frac{1}{5} du = d\theta$$

$$\frac{2\sqrt{3x}}{3} = 3dx \qquad \frac{1}{5} \int \tan u du$$

$$\frac{2\sqrt{3x}}{3} = dx \qquad = -\frac{1}{5} \ln |\cos 5\theta| + C$$

$$\frac{2\sqrt{3}}{3} \int \frac{1}{1 - \frac{1}{1}} du = \frac{2\sqrt{3}}{3} \left( u - \ln |u| \right) + C$$

$$\frac{2\sqrt{3}}{3} \int 1 - \frac{1}{1} du = \frac{2\sqrt{3}}{3} \left( u - \ln |u| \right) + C$$

$$\frac{2\sqrt{3}}{3} \int 1 + \sqrt{3x} - \ln |u| + \sqrt{3x} \right) + C$$

#### Practice

Try these problems from the textbook:

p. 338 #5,11,17,25,33,51,63

**Extra Practice with Answers**