Math Virtual Learning

Calculus AB

April 28, 2020

Lesson: Tuesday, April 28, 2020

Objective/Learning Target:

Lesson 2 Limits Review

Evaluate a limit using properties of limits

Develop and use a strategy for finding limits

Evaluate a limit using dividing out and rationalizing techniques

Properties of Limits Review

Khan Academy:
Review of Properties of Limits

THEOREM 1—Limit Laws If L, M, c, and k are real numbers and

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M, \quad \text{then}$$

1. *Sum Rule:* $\lim_{x \to c} (f(x) + g(x)) = L + M$
2. *Difference Rule:* $\lim_{x \to c} (f(x) - g(x)) = L - M$
3. *Constant Multiple Rule:* $\lim_{x \to c} (k \cdot f(x)) = k \cdot L$
4. *Product Rule:* $\lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M$
5. *Quotient Rule:* $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0$
6. *Power Rule:* $\lim_{x \to c} [f(x)]^n = L^n$, n a positive integer
7. *Root Rule:* $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer

(If n is even, we assume that $\lim_{x \to c} f(x) = L > 0$.)

Practice- Use the given information

Given that $\lim_{x \to 2} h(x) = 5$ and $\lim_{x \to 2} g(x) = 0$, find the following limits.

a. $\lim_{x \to 2}[g(x) + h(x)]$

b. $\lim_{x \to 2}[3\,h(x)]$

c. $\lim_{x \to 2}[g(x)\,h(x)]$

d. $\lim_{x \to 2} \frac{h(x)}{g(x)}$

Hint: We can rewrite each of these limits in this way. Then, use the given information.

$$\lim_{x \to 2} g(x) + \lim_{x \to 2} h(x)$$

Practice Answers

Given that $\lim_{x\to 2} h(x) = 5$ and $\lim_{x\to 2} g(x) = 0$, find the following limits.

a. $\lim_{x\to 2}[g(x) + h(x)] = 0 + 5 = 5$

b. $\lim_{x\to 2}[3\,h(x)] = 3 \cdot 5 = 15$

$= 3 \lim_{x\to 2} h(x)$

c. $\lim_{x\to 2}[g(x)\,h(x)] = 0 \cdot 5 = 0$

d. $\lim_{x\to 2} \frac{h(x)}{g(x)} = \frac{\lim_{x\to 2} h(x)}{\lim_{x\to 2} g(x)} = \frac{5}{0} \to \infty$

Finding Limits Algebraically

Evaluate the limit using direct substitution, if possible.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$$

$$\lim_{x \to 2} \frac{x^2}{3x+4}$$

Hints for **finding a limit algebraically**:

- Plug in the limiting value and see if you get a defined value.
- If you get an undefined value (zero in the denom.) try algebra (i.e. factoring) to simplify first.
- If you cannot simplify, try plugging in numbers very close to the limiting value.

Finding Limits Algebraically

$$\lim_{x\to2}\frac{x^2+x-6}{x^2-4}$$

$\frac{0}{0}$??

$$\lim_{x\to2}\frac{x^2}{3x+4} = \frac{2^2}{3(2)+4}$$

$$= \frac{4}{10} = \boxed{\frac{2}{5}}$$

Finding Limits Algebraically

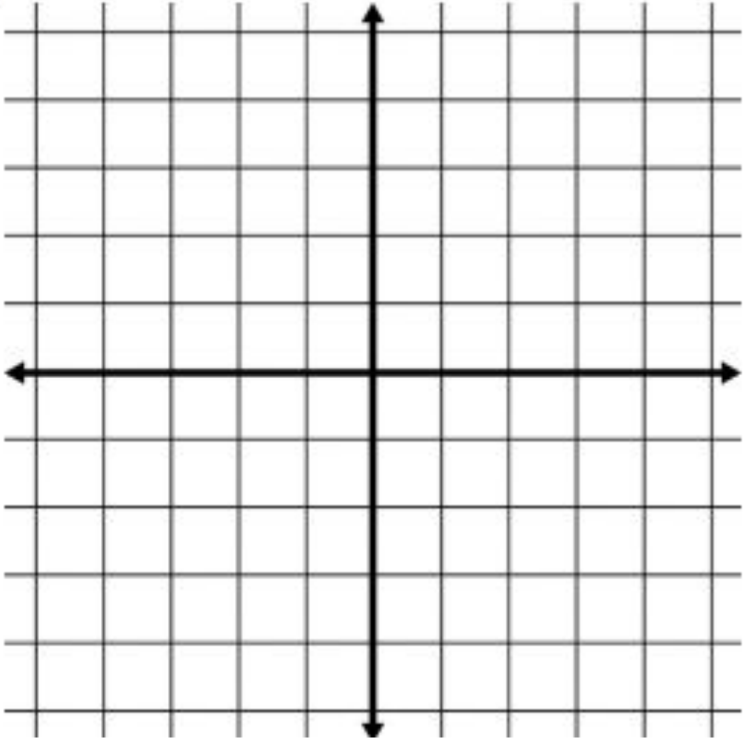
Evaluate the limit using direct substitution, if possible.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$$

Why does this happen? What does the graph look like? Let's check. Graph the function on the grid.

a. Use the Table feature on your calculator to find the y-value at x=2. ______________

b. Looking at the graph, what y-value is the function *approaching* as x gets closer and closer to x=2 from the left and the right?



Answers

Evaluate the limit using direct substitution, if possible.

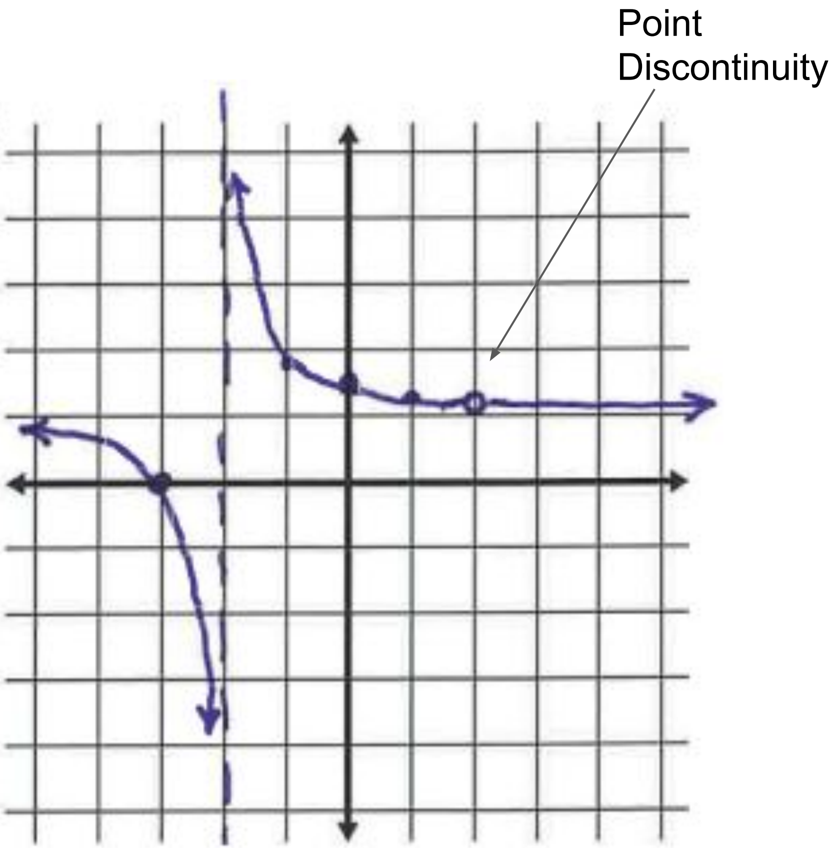
$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$$ $\frac{0}{0}$??

Why does this happen? What does the graph look like? Let's check. Graph the function on the grid.

a. Use the Table feature on your calculator to find the y-value at x=2. ERROR

b. Looking at the graph, what y-value is the function *approaching* as x gets closer and closer to x=2 from the left and the right?

$\frac{5}{4}$



Finding Limits Algebraically

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$$

a. Factor the numerator and denominator of the function. Can this expression be further simplified?

b. Now try plugging in x=2 into your simplified function. What value do you get?

Answer

a. Factor the numerator and denominator of the function. Can this expression be further simplified?

$$\frac{(x-2)(x+3)}{(x-2)(x+2)} = \frac{(x+3)}{(x+2)}$$

b. Now try plugging in x=2 into your simplified function. What value do you get?

$$\frac{5}{4}$$

Same factor in the numerator and denominator = hole in graph!

Try These

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$$

$$\lim_{x \to \pi} x \sin 2x$$

$$\lim_{x \to -7} \frac{\left(x^2 + 3x - 28\right)}{x + 7}$$

$$\lim_{x \to 3} \frac{1}{(x-3)^2}$$

$$\lim_{x \to \pi/2} (\tan x)$$

Try These

$$\lim_{x\to 3}\frac{x^2-x-6}{x-3}$$

$$\lim_{x\to 3}\frac{(x-3)(x+2)}{(x-3)}$$

$$=\lim_{x\to 3}(x+2) = 3+2 = \boxed{5}$$

$$\lim_{x\to\pi} x\sin 2x$$

$$\pi \sin 2\pi$$

$$=\pi(0)$$

$$=\boxed{0}$$

$$\lim_{x\to -7}\frac{(x^2+3x-28)}{x+7}$$

$$=\lim_{x\to -7}\frac{(x+7)(x-4)}{x+7}$$

$$=\lim_{x\to -7}(x-4) = -7-4$$

$$\boxed{-11}$$

$$\lim_{x\to 3}\frac{1}{(x-3)^2}$$

From left: $\frac{1}{(2.99-3)^2} \to +\infty$

So $\lim_{x\to 3^-} f(x) = +\infty$

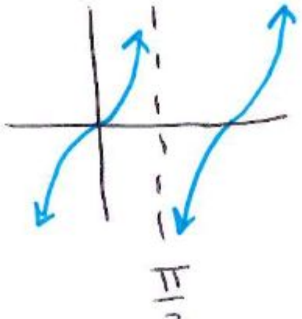
From right: $\frac{1}{(3.01-3)^2} \to +\infty$

So $\lim_{x\to 3^+} f(x) = +\infty$

Therefore $\lim_{x\to 3} f(x) = \boxed{\infty}$

$$\lim_{x\to \pi/2}(\tan x)$$

$\tan\frac{\pi}{2} \to$ DNE



Evaluating Limits Algebraically: You must know how to determine limits using various methods, including:

- Direct substitution → Plug in the limiting value, as long as the denominator is not zero

  EXAMPLE: $\lim_{x \to 0} \frac{5}{x-2} = \frac{5}{0-2} = -\frac{5}{2}$

- Algebraic manipulation → Try to factor, FOIL, or simplify the expression

  EXAMPLE: $\lim_{x \to 2} \frac{x^2+5x-14}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+7)}{x-2} = \lim_{x \to 2}(x+7) = 9$

- Numerical analysis → Plug in values very close to the limiting value.

  EXAMPLE: $\lim_{x \to 1} \frac{4}{x-1}$ Check left and right hand limits:

  LEFT: $\lim_{x \to 1^-} \frac{4}{x-1}$ $\quad \frac{4}{0.99-1} = \frac{4}{-.01} = -400 \to -\infty$

  RIGHT: $\lim_{x \to 1^+} \frac{4}{x-1}$ $\quad \frac{4}{1.01-1} = \frac{4}{.01} = 400 \to +\infty$

  → Therefore, $\lim_{x \to 1} \frac{4}{x-1}$ DNE

Remember, we don't care if the function is not defined <u>at</u> the limiting value. We only care about the function behavior as x gets <u>close</u> to the limiting value.

A little more practice

Evaluate $\lim_{x \to -2} \frac{x^2 - 4x - 12}{x+2}$

A little more practice- answer

Evaluate $\lim_{x \to -2} \frac{x^2 - 4x - 12}{x+2}$

$$\lim_{x \to -2} \frac{(x+2)(x-6)}{(x+2)} = \lim_{x \to -2} (x-6) = -8$$

Challenge- Multiply by the Conjugate

2. Evaluate $\lim_{x \to 4} \frac{4-x}{2-\sqrt{x}}$ $\quad \frac{0}{0}$?

$$\lim_{x \to 4} \frac{4-x}{2-\sqrt{x}} = \lim_{x \to 4} \frac{(2-\sqrt{x})(2+\sqrt{x})}{(2-\sqrt{x})} = 4$$

Method 1: Factor

OR

$$\lim_{x \to 4} \left(\frac{4-x}{2-\sqrt{x}} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} \right) = \lim_{x \to 4} \frac{(4-x)(2+\sqrt{x})}{4-x} = 4$$

Method 2: Multiply by the conjugate

Notice: there are 2 different methods for solving. Method 1, factoring, works like the previous examples. However, this method of factoring is not always obvious. We often forget 4 - x is a *difference of 2 squares.*

Method 2 takes the conjugate pair (same terms but the opposite sign) and creates the *difference of squares* factors. Be sure to multiply both the numerator and denominator by this *conjugate factor*.

Try it

Find $\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} =$

Answer

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} \to \frac{0}{0}$$

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

$$= \lim_{x \to 9} \frac{(x-9)(\sqrt{x}+3)}{x-9}$$

$$= \lim_{x \to 9} \sqrt{x}+3 = \boxed{6}$$

Practice

Practice with Answers (pgs. 5-6)

Textbook Suggested Problems

Pg. 67: 3, 10, 18, 26, 28, 34, 38, 42-52 even, 60, 68, 72, 84, 113-117