Inspiring Greatness
INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

Calculus AB

Review of Integration and The Fundamental Theorem of Calculus

May 12, 2020

Lesson: Tuesday, May 12, 2020

Objective/Learning Target:

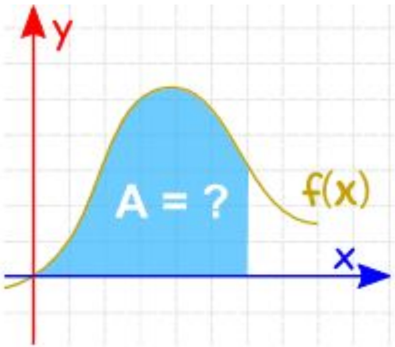
Use Basic Integration Rules

Practice definite and indefinite integrals

Understand the Fundamental Theorem of Calculus

Integration

Integration can be used to find areas, volumes, central points and many useful things. But it is often used to find the **area underneath the graph of a function** like this:



The integral of many functions are well known, and there are useful rules to work out the integral of more complicated functions, many of which are shown here.

Definition of an Antiderivative

A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

A similar table of derivative and integral rules can be found in the front of your textbook.

Basic Integration Rules

Differentiation Formula	*Integration Formula*	
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$	
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$	
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$	
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	Power Rule
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$	
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$	
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$	
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$	
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$	
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$	

Try these.

Evaluate each indefinite integral.

a. $\int 7dx$

b. $\int (3x^2 - 4)dx$

c. $\int 4e^x dx$

d. $\int \frac{x^3+5}{x^2} dx$

e. $\int \csc^2(x)\, dx$

Remember: An integral function is a way to ask yourself, “what function gave this derivative?”

Don’t forget your “constant of integration”!

Try it answers.

1. Evaluate each indefinite integral.

a. $\int 7dx$

$7x + C$

b. $\int (3x^2 - 4)dx$

$x^3 - 4x + C$

c. $\int 4e^x dx$

$4e^x + C$

d. $\int \frac{x^3+5}{x^2} dx$

$\int (x + 5x^{-2})dx$

$\frac{x^2}{2} - 5x^{-1} + C$

e. $\int \csc^2(x)\, dx$

$-\cot x + C$

Given an initial condition.

2. Find a function, $f(x)$, whose derivative is given by $f'(x) = 4x^3 - 2x^2 + 7x + 1$ and passes through the point $(3, 8)$.

Because we are given a value of the function we can now evaluate to find out the "+ c" value.

Answer

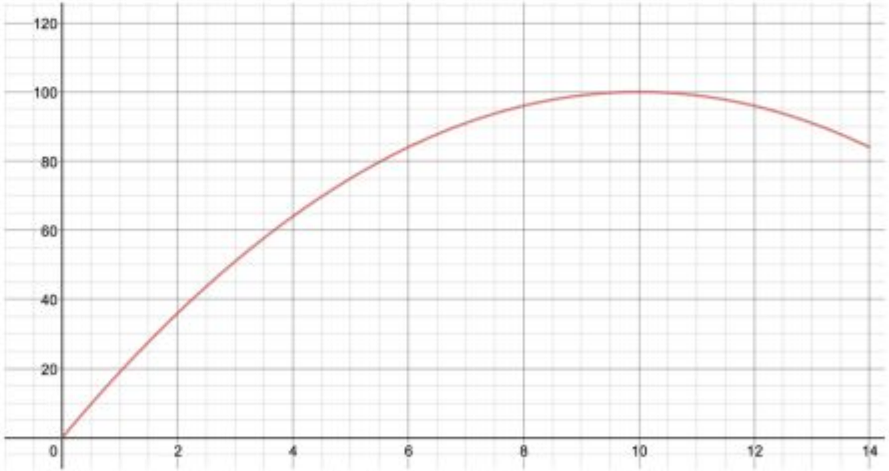
2. Find a function, $f(x)$, whose derivative is given by $f'(x) = 4x^3 - 2x^2 + 7x + 1$ and passes through the point $(3, 8)$.

$$f(x) = x^4 - \frac{2}{3}x^3 + \frac{7}{2}x^2 + x + C$$

$$f(3) = 8 \Rightarrow \quad 8 = 97.5 + C \Rightarrow C = -89.5$$

$$f(x) = x^4 - \frac{2}{3}x^3 + \frac{7}{2}x^2 + x - 89.5$$

You have just taken over as manager of a struggling umbrella company. Umbrellas are manufactured at a rate given by $G(t) = 20t - t^2$ umbrellas per hour for $0 \le t \le 14$, and t represents hours after the factory opens in the morning (6 AM). $G(t)$ is graphed below.



1. After a leisurely breakfast, you arrive at work at 9 AM.
   a. Write an expression that gives the number of umbrellas that have been produced before you even arrived.
   b. Roughly how many umbrellas were produced during this time?

Remember, the first derivative gives a rate of change. When you are given a rate, you can use an antiderivative to find the number of umbrellas at a certain time.

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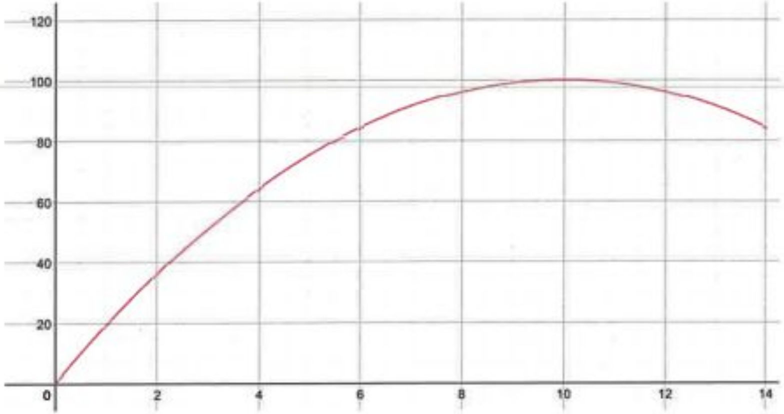


1. After a leisurely breakfast, you arrive at work at 9 AM.
   a. Write an expression that gives the number of umbrellas that have been produced before you even arrived.

   $\int_0^3 G(t)\,dt$

   b. Roughly how many umbrellas were produced during this time?

   (Answers will vary based on method)

   $\frac{G(3)+G(0)}{2} \cdot 3 = \frac{51}{2} \cdot 3 = 76.5$

   roughly 77 umbrellas

2. At 12:30 PM you break for a long lunch.
    a. Write an expression that gives the number of umbrellas that have been produced that day up until your lunch break.
    b. Roughly how many umbrellas were produced during this time?

3. Write an equation involving an integral for a function $f(x)$, that gives the number of umbrellas produced x hours after the factory has opened.

4. Find $f'(8)$ and interpret your answer in the context of this problem.

5. When is $f(x)$ changing the fastest? How do you know?

2. At 12:30 PM you break for a long lunch.

   a. Write an expression that gives the number of umbrellas that have been produced that day up until your lunch break.

   $$\int_0^{6.5} G(t)\,dt$$

   b. Roughly how many umbrellas were produced during this time?

   (Answers will vary based on method)

   $$76.5 + \frac{G(3)+G(6.5)}{2}\cdot 3.5 = 319 \text{ umbrellas}$$

3. Write an equation involving an integral for a function $f(x)$, that gives the number of umbrellas produced x hours after the factory has opened.

   f(x) is an accumulation function!

   $$f(x) = \int_0^{x} G(t)\,dt$$

4. Find $f'(8)$ and interpret your answer in the context of this problem.

   $f'(8) = G(8)$

   $\frac{d}{dx}(f(x)) = G(x)$

   $f'(8)$ is the rate of change of umbrellas produced at $t=8$ $G(8) = 20(8) - 8^2 = 96$ umbrellas per hour

5. When is $f(x)$ changing the fastest? How do you know?

   At $t=10$ (4 PM) because the rate of production ($f'(x)/g(t)$) is at a maximum.

FTC

The Fundamental Theorem of Calculus

Important Ideas:

An accumulation function outputs the area under a curve from some starting value to x (the input).

$$F(x) = \int_{c}^{x} f(t)\,dt$$

independent variable → x; rate of accumulation → f(t); c ← starting value

FTC (Part 1): Rate of change of an accumulation function

$$\frac{d}{dx}\int_{c}^{x} f(t)\,dt = \frac{d}{dx}F(x) = f(x)$$

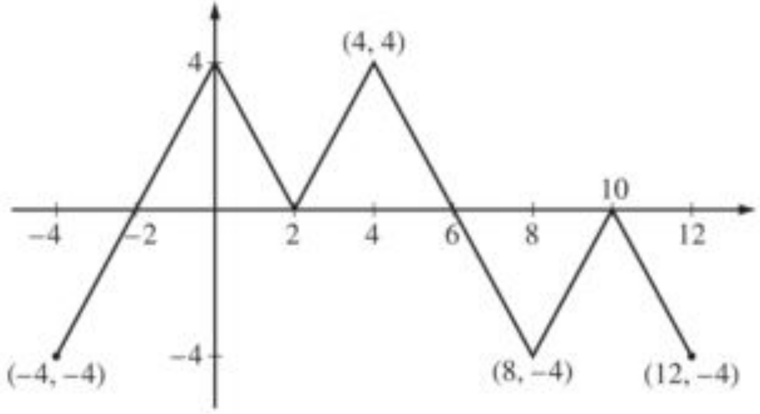
Practice FTC

1. The graph of $f(t)$ is shown below. Let $h(x) = \int_{-2}^{x} f(t)dt$.

a. Find $h(2)$.

b. Find $h(8)$.

c. Find $h'(x)$.



Graph of f

Answers

1. The graph of $f(t)$ is shown below. Let $h(x) = \int_{-2}^{x} f(t)dt$.

a. Find $h(2)$.

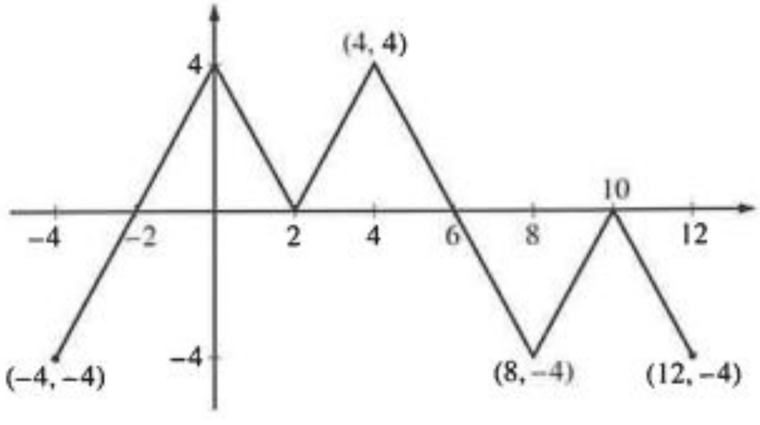
$$\int_{-2}^{2} f(t)dt = \frac{1}{2}(4)(4) = 8$$

b. Find $h(8)$.

$$\int_{-2}^{8} f(t)dt = 8 + 8 - \frac{1}{2}(2)(4)$$
$$= 12$$

c. Find $h'(x)$.

$$h'(x) = f(x)$$



Graph of f

Practice

Definite Integral Practice

Basic Integrals Practice

More Examples with Answers