Inspiring Greatness
INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

Calculus AB

Review of the Chain Rule for Derivatives

May 5, 2020

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Lesson: Tuesday, May 5, 2020

Objective/Learning Target:
Lesson 2 Derivatives Review
Find the derivative of a composite function using the chain rule
Use the chain rule with trig functions

Introduction

[Intro to the Chain Rule]

[Composite Function Review]

Chain Rule Formula

Chain Rule

If f and g are both differentiable and $F(x)$ is the composite function defined by $F(x) = f(g(x))$ then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))\, g'(x)$$

Differentiate outer function

Differentiate inner function

[Khan Academy Example]

More Examples

Find the derivatives of each of the following

a) $f(x) = (2x^3 + 7)^6$

b) $f(x) = \dfrac{3}{(x^2 - 3)^2}$

c) $f(x) = \sqrt{x^2 + 1}$

[Another Khan Academy Example]

Solution:

a) $f'(x) = 6(2x^3 + 7)^5 (6x^2)$

b) $f(x) = \dfrac{3}{(x^2 - 3)^2} = 3(x^2 - 3)^{-2}$

$f'(x) = 6(x^2 - 3)^{-3}(2x)$

$= \dfrac{-12x}{(x^2 - 3)^3}$

c) $f(x) = \sqrt{x^2 + 1} = (x^2 + 1)^{\frac{1}{2}}$

$f'(x) = \dfrac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(2x)$

$= \dfrac{x}{\sqrt{x^2 + 1}}$

Trig Examples with the Chain Rule

$$\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$$

$$\frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$$

$$\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$$

$$\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$$

$$\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$$

$$\frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$$

All trig formulas for derivatives

Khan Academy Trig Function Example 1

Khan Academy Trig Function Example 2

Review of Product and Quotient Rule

Example:

Differentiate $y = x^2 \sin x$

Solution:

Using the Product Rule and the sin derivative, we have

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}\left(x^2\right)$$
$$= x^2 \cos x + 2x \sin x$$

Example:

Differentiate $f(x) = \dfrac{\sec x}{1+\tan x}$

Solution:

Using the Quotient Rule and the sec and tan derivative, we have

$$f'(x) = \frac{(1+\tan x)\frac{d}{dx}(\sec x) - \sec x \frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$
$$= \frac{(1+\tan x)\sec x \tan x - \sec x \cdot \sec^2 x}{(1+\tan x)^2}$$
$$= \frac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2}$$
$$= \frac{\sec x(\tan x - 1)}{(1+\tan x)^2} \quad (\text{use the identity } \tan^2 x + 1 = \sec^2 x)$$

Ap Examples

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

Solution

(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

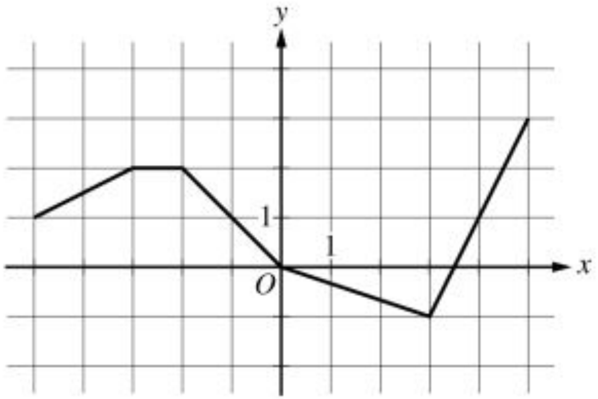
(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.

AP Example

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

AP Example Answer

(a) $f'(x) = -2\sin(2x) + \cos x \, e^{\sin x}$

$f'(\pi) = -2\sin(2\pi) + \cos\pi \, e^{\sin\pi} = -1$

(b) $k'(x) = h'(f(x)) \cdot f'(x)$

$k'(\pi) = h'(f(\pi)) \cdot f'(\pi) = h'(2) \cdot (-1)$
$= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$

(c) $m'(x) = -2g'(-2x) \cdot h(x) + g(-2x) \cdot h'(x)$

$m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2)$
$= -2(-1)\left(-\frac{2}{3}\right) + 5\left(-\frac{1}{3}\right) = -3$

(d) g is differentiable. $\Rightarrow$ g is continuous on the interval $[-5, -3]$.

$\frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$

Therefore, by the Mean Value Theorem, there is at least one value c, $-5 < c < -3$, such that $g'(c) = -4$.

Bonus MVT Review

More Practice

Chain Rule Practice

Chain Rule Practice with Trig Functions

Higher Order Derivatives Practice

Textbook Practice- Pg. 137: 1-29 by 4, 41-73 by 4, 83, 89, 91-97, 103, 111